Homework Solutions

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$$\theta_B = \frac{1.333}{1}, \quad \theta_B = 53.12^o$$

If unpolarized light is incident at this angle, the reflected beam is polarized with its electric field parallel to the surface of reflection. This will depend of λ since in general $n = n(\lambda)$ as we say in section 20.2.

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Position "B" will let no light pass. The reflected ray has its electric field polarized parallel to the "A" arrow.

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$$frac = \frac{\int_0^{\theta_c} \sin\theta d\theta \int_0^{2\pi} d\phi}{\int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi} = \frac{1}{2}\Big(-\cos\theta\Big|_0^{\theta_c}\Big) = \frac{1}{2}\Big(1-\cos\theta_c\Big) = \frac{1}{2}\Big(1-\sqrt{1-n^{-2}}\Big)$$

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$$\sin\psi = \frac{h}{R}, \quad n\sin\psi = 1\sin 90 = 1, \quad n\frac{h}{R} = 1, \quad h = \frac{R}{n}$$

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The electric field amplitude of the incident light is E_0, $\bar{I}_0 = 100\frac{W}{m^2} = \frac{1}{2\mu_0 c}E_0^2$. The electric field amplitude of the light transmitted through the first junction is

$$E_T = \frac{2\cdot 1}{1+3}E_0 = \frac{2}{4}E_0$$

and transmitted through the second junction

$$E_{TT} = \frac{2\cdot 3}{3+1}E_T = \frac{6}{4}\frac{2}{4}E_0 = \frac{12}{16}E_0 = \frac{3}{4}E_0$$

The intensity transmitted through the sheet into the air beyond is

$$\bar{I}_{TT} = \frac{1}{2\mu_0 c}E_{TT}^2 = \frac{3^2}{4^2}\frac{1}{2\mu_0 c}E_0^2 = 56.25\frac{W}{m^2}$$

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$$100 = \frac{1}{2\mu_0 c}E_{inc}^2, \qquad I_{trans} = \frac{1}{2\mu_0 c}\Big(E_{inc}\frac{2\cdot 1}{1+3}\frac{2\cdot 3}{1+3}\Big)^2 = 0.56\cdot\frac{1}{2\mu_0 c}E_{inc}^2 = 56\frac{W}{m^2}$$

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$$E_{dir} = E_{inc}\frac{2\cdot 1}{1+3}\frac{2\cdot 3}{1+3} = \frac{12}{16}E_{inc}, \qquad E_{sec} = -E_{inc}\frac{2\cdot 1}{1+3}\frac{2\cdot 3}{1+3}\Big(\frac{3-1}{3+1}\Big)^2 = -\frac{3}{16}E_{inc}$$

$$I_{trans} = \frac{1}{2\mu_0 c}\Big(\frac{12}{16}E_{inc} - \frac{3}{16}E_{inc}\Big)^2 = 0.316\,\frac{1}{2\mu_0 c}E_{inc}^2 = 31.6\frac{W}{m^2}$$

Clearly there will be many such beams that need to be taken into account to find the total, true transmission through the glass.

The two rays are 180^o out of phase after two reflections with no inversions if

$$2\cdot 3D = (m+\frac{1}{2})\lambda, \quad D = \frac{\lambda}{2\cdot 2\cdot 3} = 4.17\times 10^{-8}\,m$$

is the minimal glass thickness.

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$$1\sin\theta = n\sin\phi, \quad \theta\approx n\phi, \quad x = D\tan\phi \approx D\phi, \quad x = d\tan\theta\approx d\theta$$

$$1 = \frac{D}{d}\frac{\phi}{\theta} = \frac{D}{d}\frac{1}{n}, \quad d \approx \frac{D}{n}$$

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$$\sin\theta = \frac{L}{R}, \quad n\sin\psi = 1\sin\theta, \quad \sin\psi = \frac{L}{nR}$$

but

$$2\psi + 180 - \theta = 180, \quad \psi = \frac{\theta}{2}$$

therefore

$$\sin\theta = \frac{L}{R} = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2\frac{L}{nR}\sqrt{1-(\frac{L}{nR})^2}$$

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$$\Big(\mathbf{p} - 2(\mathbf{p}\cdot\mathbf{n})\,\mathbf{n}\Big)\cdot\Big(\mathbf{p} - 2(\mathbf{p}\cdot\mathbf{n})\,\mathbf{n}\Big) = \mathbf{p}\cdot\mathbf{p} - 4\mathbf{p}\cdot\Big((\mathbf{p}\cdot\mathbf{n})\,\mathbf{n}\Big) + 4\Big((\mathbf{p}\cdot\mathbf{n})\,\mathbf{n}\Big)\cdot\Big((\mathbf{p}\cdot\mathbf{n})\,\mathbf{n}\Big)$$
$$= p^2 - 4(\mathbf{p}\cdot\mathbf{n})^2 + 4(\mathbf{p}\cdot\mathbf{n})^2\,n^2 = 1 - 4(\mathbf{p}\cdot\mathbf{n})^2 + 4(\mathbf{p}\cdot\mathbf{n})^2\ \cdot 1 = 1$$

since $p^2 = n^2$ since they are ubnit vectors.
For this corner, $\mathbf{n}_1 = -\sin\theta\mathbf{j} + \cos\theta\mathbf{i}$ (right surface) and $\mathbf{n}_2 = +\sin\phi\mathbf{j} + \cos\phi\mathbf{i}$ (left surface). Let the ray hit the right surface first. After reflection

$$\mathbf{p} = -\mathbf{i} \to -\mathbf{i} - 2(-\cos\theta)\,(-\sin\theta\mathbf{j} + \cos\theta\mathbf{i}) = \cos(2\theta)\mathbf{i} - \sin(2\theta)\mathbf{j} = \mathbf{p}'$$

now let it hit the second surface;

$$\mathbf{p}' \to \mathbf{p}' - 2(\mathbf{p}'\cdot\mathbf{n}) = \cos(2\theta)\mathbf{i} - \sin(2\theta)\mathbf{j} - 2(\cos(2\theta)\cos\phi - \sin(2\theta)\sin\phi)\,(\sin\phi\mathbf{j} + \cos\phi\mathbf{i})$$
$$= -\cos(2\theta - 2\phi)\mathbf{i} - \sin(2\theta - 2\phi)\mathbf{j}$$

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The three transformations

$$\mathbf{p} \to \mathbf{p} - 2(\mathbf{i}\cdot\mathbf{p})\mathbf{i}, \quad \mathbf{p} \to \mathbf{p} - 2(\mathbf{j}\cdot\mathbf{p})\mathbf{j}, \quad \mathbf{p} \to \mathbf{p} - 2(\mathbf{k}\cdot\mathbf{p})\mathbf{k}$$

applied to $\mathbf{p} = (p_x, p_y, p_z)$ in succession perform

$$(p_x, p_y, p_z) \to (-p_x, p_y, p_z) \to (-p_x, -p_y, p_z) \to (-p_x, -p_y, -p_z)$$

and therefore completely invert the propagation vector.

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This is simply a repeat of problem **254**.

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The image of the closest end is at $\mathcal{I} = \frac{\mathcal{O}f}{\mathcal{O}-f}$, the other end, a distance $\mathcal{O} + \ell$ from the mirror, has its image at $\mathcal{I}' = \frac{(\mathcal{O}+\ell)f}{\mathcal{O}+\ell-f}$, and the distance between the two images is the length of the image along the optical axis;

$$\ell' = |\mathcal{I}' - \mathcal{I}| = \frac{\mathcal{O}f}{\mathcal{O}-f} - \frac{(\mathcal{O}+\ell)f}{\mathcal{O}+\ell-f}$$
$$\approx \frac{\mathcal{O}f}{\mathcal{O}-f} - \frac{(\mathcal{O}+\ell)f}{\mathcal{O}-f}\Big(1 - \frac{\ell}{\mathcal{O}-f} + \cdots\Big) \approx \ell\Big(\frac{f}{\mathcal{O}-f}\Big)^2 + \cdots$$

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The object moves at $v_O = \frac{d\mathcal{O}}{dt}$, the image is at

$$\frac{1}{\mathcal{I}} = \frac{2}{R} - \frac{1}{\mathcal{O}}, \quad \mathcal{I} = \frac{\mathcal{O}R}{2\mathcal{O}-R}$$

so just differentiate this and use $v_I = \frac{d}{dt}\mathcal{I}$.

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$$\frac{1}{f} = (1.5-1)\Big(\frac{1}{\infty} + \frac{1}{20}\Big), \qquad f = 40, \quad \frac{1}{40} + \frac{1}{\mathcal{I}} = \frac{1}{40}, \qquad \mathcal{I} = \infty$$

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The image formed by the first lens

$$\frac{1}{40} + \frac{1}{\mathcal{I}_1} = \frac{1}{20}, \qquad \mathcal{I}_1 = 40$$

lies to the right of the second lens a distance $30\,cm$, and becomes a virtual object of object distance $\mathcal{O}' = -30$. The final image is formed at

$$\frac{1}{I_f} = \frac{1}{-15} - \frac{1}{-30}, \qquad \mathcal{I}_f = -30$$

measured from the second lens.

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If it is a real image, then it is a converging lens $f > 0$. Furthermore the image is inverted, and $-\frac{\mathcal{I}}{\mathcal{O}} = -\frac{1}{2}$, then $\mathcal{O} + \mathcal{I} = 40$, substitute to get $\mathcal{O} = \frac{80}{3}$, $\mathcal{I} = \frac{40}{3}$, $\frac{1}{f} = \frac{3}{40} + \frac{3}{80} = \frac{9}{80}$ so $f = \frac{80}{9}$.

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The critical idea is that the image from the first lens becomes the object for the second;

$$\frac{1}{\mathcal{O}} + \frac{1}{\mathcal{I}_1} = \frac{1}{f_1}, \quad \frac{1}{\mathcal{I}_1} = \frac{1}{f_1} - \frac{1}{\mathcal{O}}$$

and now $\mathcal{I}_1 = \mathcal{O}_2$ since there is no space between the lenses, then

$$\frac{1}{\mathcal{O}_2} + \frac{1}{\mathcal{I}_2} = \frac{1}{f_2}, \quad \frac{1}{\mathcal{I}_2} = \frac{1}{f_2} - \frac{1}{\mathcal{O}_2} = \frac{1}{f_2} - \Big(\frac{1}{f_1} - \frac{1}{\mathcal{O}}\Big)$$

or

$$\frac{1}{\mathcal{O}} + \frac{1}{\mathcal{I}_2} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{\frac{f_1 f_2}{f_1 + f_2}}$$

280

If it is a real image, then it is a converging lens $f > 0$. Furthermore the image is inverted, $D = \mathcal{O} + \mathcal{I}$,

$$\frac{1}{D - \mathcal{I}} + \frac{1}{\mathcal{I}} = \frac{1}{f}, \qquad \mathcal{I}^2 - \mathcal{I}D + Df = 0$$

The two image locations are

$$\mathcal{I}_{1,2} = \frac{D \pm \sqrt{D^2 - 4Df}}{2}$$

and they are seperated by

$$\mathcal{I}_2 - \mathcal{I}_1 = \sqrt{D^2 - 4Df}$$

Note that $\mathcal{I}_1 = \frac{D - \sqrt{D^2 - 4Df}}{2} = \frac{D-d}{2}$ and $\mathcal{O}_1 = D - \mathcal{I}_1 = \frac{D+d}{2} = \mathcal{I}_2$, and also $\mathcal{O}_2 = D - \mathcal{I}_2 = \mathcal{I}_1$! The image height ratio is the magnification ratio

$$\frac{m_2}{m_1} = \frac{-\frac{\mathcal{I}_2}{\mathcal{O}_2}}{-\frac{\mathcal{I}_1}{\mathcal{O}_1}} = \frac{-\frac{\mathcal{O}_1}{\mathcal{I}_1}}{-\frac{\mathcal{I}_1}{\mathcal{O}_1}} = \frac{\mathcal{O}_1^2}{\mathcal{I}_1^2} = \Big(\frac{D+d}{D-d}\Big)^2$$

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This is the same as 260, just put $D = 100$ and $f = +16$ into the image formula.

$$\frac{1}{\mathcal{O}} + \frac{1}{100 - \mathcal{O}} = \frac{1}{+16}, \quad \mathcal{O}^2 - 100\mathcal{O} + 1600 = 0, \qquad \mathcal{O} = 80, \quad 20$$

f must be positive to form real image.

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$$\frac{1}{\mathcal{O}} + \frac{1}{-30} = \frac{2}{R}, \quad \frac{1}{\mathcal{O}} + \frac{1}{-10} = \frac{2}{-R}, \qquad \mathcal{O} = 15, \quad R = +60$$