Physics 202 EXAM I Feb. 19, 2008

Instructions Your Name;

You must show all work to get full credit, and must give a numerical answer in response to questions in which numerical data is given. Neatness counts in the sense that it helps me to allot partial credit.

Problem 1. 18 points (P-30)

A pipe used as a musical instrument can create resonant sound waves of frequency $680\,Hz$ and $1020\,Hz$. These are both standing waves within the pipe. In addition there are **two** resonant wave with a frequency between these two values. The speed of sound in air is $340\,\frac{m}{s}$.

A. 6 points Determine the length of the pipe, and whether or not it is closed at one end or open at both.
If the pipe is closed, $L = (2n+1)\frac{\lambda}{4}$, if it is open $L = 2n\frac{\lambda}{4}$, in both cases frequency increases with n

$$680 = \frac{c}{4L}(2n+1), \quad 1020 = \frac{c}{4L}(2(n+3)+1) \quad \text{has no integer solutions}$$

$$680 = \frac{c}{4L}(2n), \quad 1020 = \frac{c}{4L}(2(n+3)) \quad \text{has integer solution} \quad n = 6$$

so the pipe is open, $L = 1.5\,m$
B. 6 points Determine the lowest frequency resonant standing wave that the pipe can support, and draw a picture of the air displacement within the pipe.

from above $f_0 = \frac{680}{6}$. A node in the middle, antinodes at/near both ends.
C. 6 points Determine the wavelengths of both the waves.

$$\lambda_1 = \frac{340}{680} = 0.5\,m, \quad \lambda_2 = \frac{340}{1020} = 0.333\,m$$

Problem 2. 18 pts

A. 6 points (P-31) A bird flying due north (against a $5\frac{m}{s}$ wind) at speed v lets out a chirp of frequency f_0. As the bird approaches you, you hear its chirp at $714.25\,Hz$. As it passes you and recedes, you hear the chirp shift down to $631.77\,Hz$. The speed of sound is $343\,\frac{m}{s}$
How fast is the bird traveling? If the bird were at rest, what would be the frequency f_0 of the chirp?

$$f_{approac} = 714.26 = \frac{f_0}{1 - \frac{v}{343-5}}, \quad F_{rec} = 631.77 = \frac{f_0}{1 + \frac{v}{343+5}}$$

$$\frac{714.25}{631.77} = \frac{1 + \frac{v}{348}}{1 - \frac{v}{338}}, \quad v \approx 21.0\frac{m}{s}$$

$$f_0 = 714.25(1 - \frac{21.0}{343}) = 670.5Hz$$

B. 6 points (P-28) A bat flies towards a mosquito hovering motionless in mid-air. The bat emits a $19,000\,Hz$ squeak that echoes from the mosquito and returns to the bat as $20,000\,Hz$. Determine the air-speed of the bat.

The bug hears

$$f_{obs} = 19,000Hz\frac{1}{1 - \frac{v}{343}}$$

This bounces off of the bug and the bat hears

$$20,000 = f_{bat} = f_{obs}(1 + \frac{v}{343}) = 19,000Hz\frac{1 + \frac{v}{343}}{1 - \frac{v}{343}} \approx 19,000\,(1 + 2\frac{v}{343} + \cdots), \quad v \approx 9.0\frac{m}{s}$$

C. 6 points (P-39) An out of control skate-boarder careens directly towards a brick wall at $8\,\frac{m}{s}$. His friend, who is standing next to the wall emits a $3000\,Hz$ whistle, what frequency is heard by the skate-boarder?

$$f = 3000Hz(1 + \frac{8}{343})$$

Problem 3. 28 points. (E-38)

The electrostatic potential at a point $(x, y, 0)$ in the xy-plane is given (in Volts V) by (distances in meters)

$$V(x,y) = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\sqrt{x^2+y^2}\right)$$

A. (E-38) 5 points. Find the $x-$component of the electric field at $(x, y, 0) = (2.0, \ ,2.0, \ 0)$.

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\lambda}{2\pi\epsilon_0} \frac{x}{x^2+y^2}$$

B. 5 points. The $1.5\,V$ equipotential is actually a circle. Find its radius. $\lambda = 1.0 \times 10^{-10} \frac{C}{m}$.

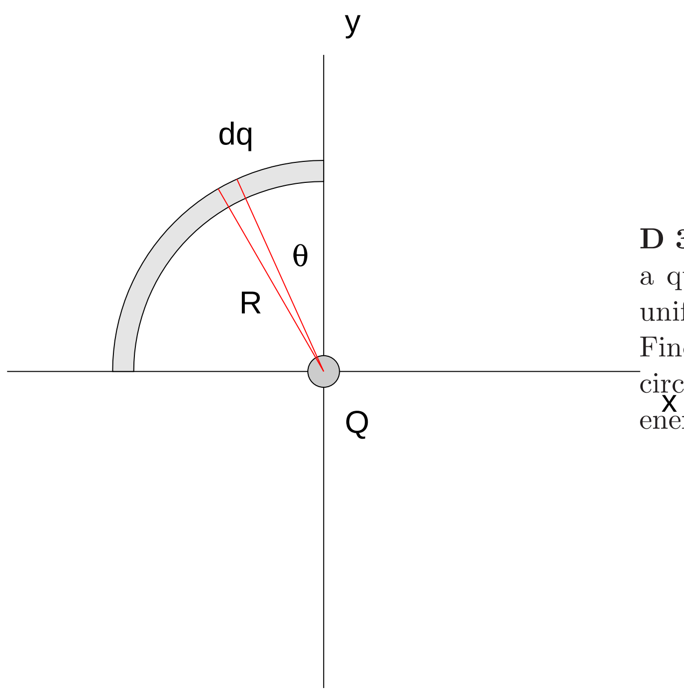
$$e^{2\frac{2\pi\epsilon_0 V}{\lambda}} = x^2 + y^2 = R^2, \quad R = e^{\frac{1.5}{1.8}} = 2.3\,m$$

C. (E-39, P-56) 6 points. An electric field

$$\mathbf{E}(\mathbf{r}) = \frac{Q}{4\pi\epsilon_0 R^3}\,\mathbf{r}, \qquad \mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$$

exists in some region of space. Q and R are constants. Find the work done in moving a charge q from $\mathbf{r}_i = 0\,\mathbf{i} + \frac{R}{2}\,\mathbf{j} + 0\,\mathbf{k}$ to $\mathbf{r}_f = 0\,\mathbf{i} + R\,\mathbf{j} + 0\,\mathbf{k}$.
You are moving it along a radial line;

$$\Delta W = q\Delta V = -q\int_{R_0}^{R_f} \frac{Q}{4\pi\epsilon_0 R^3}\, r\, dr = -q\frac{Q}{8\pi\epsilon_0 R^3}(R_f^2 - R_0^2) = -q\frac{Q}{8\pi\epsilon_0 R^3}(R^2 - (\frac{R}{2})^2) = -q\frac{3Q}{34\pi\epsilon_0 R}$$

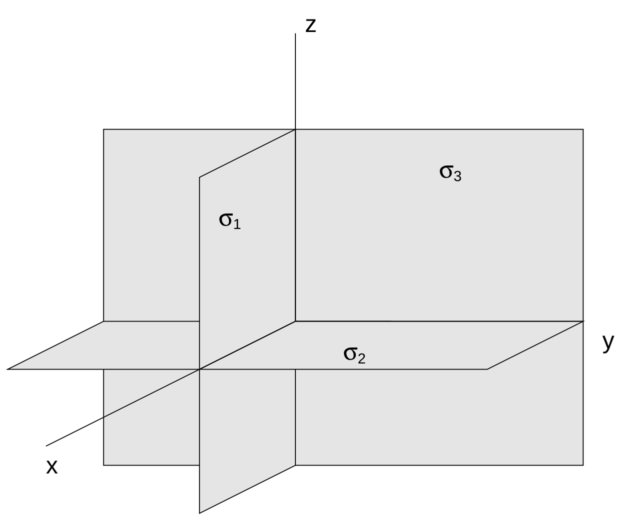


D 3+3 (P-49) A thin glass rod is bent into a quarter-circle of radius R. A charge q is uniformly distributed over it.
Find the electric field at the center of the circle. What is the electrostatic potential energy of Q?

$$|d\mathbf{F}_Q| = \frac{|kQ\,dq|}{R^2}\Big(\sin\theta\,\mathbf{i} - \cos\theta\,\mathbf{j}\Big)$$

$$dq = \frac{q}{\pi R}\,R\,d\theta = q\frac{d\theta}{\pi}$$

$$\mathbf{F}_Q = \int_0^{\frac{\pi}{2}} \frac{|kQq|}{\pi R^2}\Big(\sin\theta\,\mathbf{i} - \cos\theta\,\mathbf{j}\Big)\,d\theta = \frac{|kQq|}{\pi R^2}\,(\mathbf{i} - \mathbf{j})$$



E. 3+3 (E-40/41, P-52) Compute the electric field at point (a, b, c), $a, b, c > 0$ in the positive octant for the three intersecting infinite planes illustrated here, with $\sigma_i > 0$, $i = 1, 2, 3$. Determine the work done in moving Q from the origin to this point.

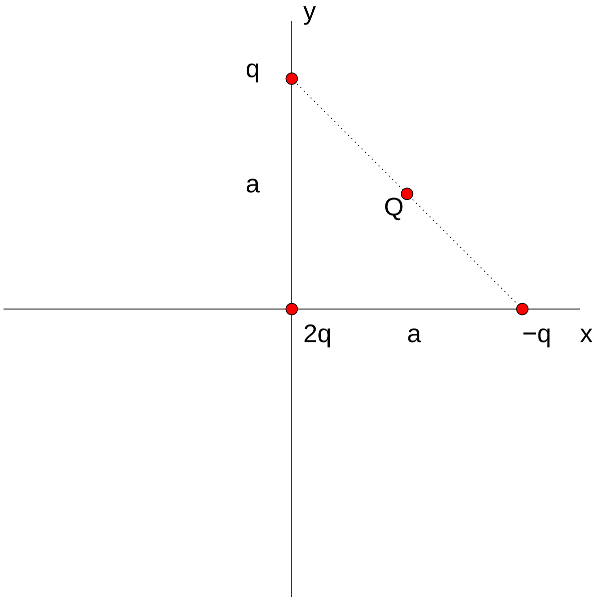
$$\mathbf{E} = 2\pi k\Big(\sigma_1\mathbf{j} + \sigma_2\mathbf{k} + \sigma_3\mathbf{i}\Big)$$

Parameterize; $\mathbf{r} = at\mathbf{i} + bt\mathbf{j} + ct\mathbf{k}$, $\mathbf{v} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$

$$W = -Q\int_0^1 2\pi k\Big(\sigma_1\mathbf{j}+\sigma_2\mathbf{k}+\sigma_3\mathbf{i}\Big)\cdot\Big(a\mathbf{i}+b\mathbf{j}+c\mathbf{k}\Big)dt$$

$$= -2\pi kQ\Big(\sigma_1 b + \sigma_2 c + \sigma_3 a\Big)$$

Problem 4. 18 points

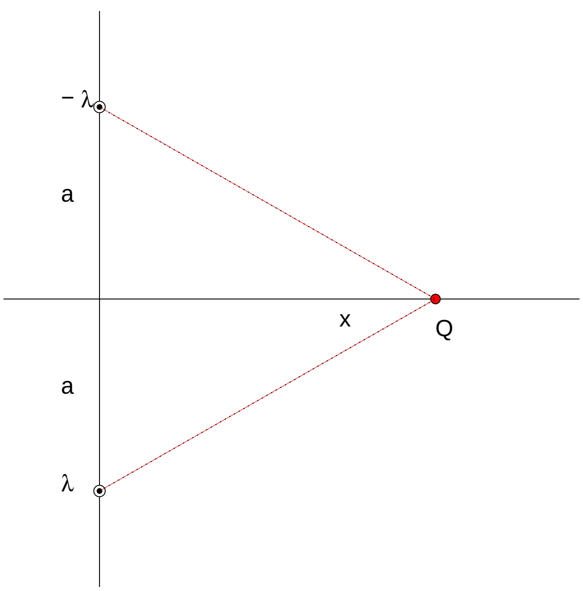


A . 4+2 (P-44)

Compute the direction and magnitude of the electric field at the location of the charge Q, and the force on Q. Compute the electrostatic potential energy of Q in the field made by the other charges.
The potential energy is trivial; $U = Q\frac{k(q-q+2q)}{\frac{a}{\sqrt{2}}}$

$$\mathbf{E} = \frac{2qk}{(\frac{a}{\sqrt{2}})^2}(\frac{\mathbf{i}}{\sqrt{2}} + \frac{\mathbf{j}}{\sqrt{2}}) + 2\frac{qk}{(\frac{a}{\sqrt{2}})^2}(\frac{\mathbf{i}}{\sqrt{2}} - \frac{\mathbf{j}}{\sqrt{2}})$$

$$\mathbf{F} = 2\,Q\,\frac{2qk}{(\frac{a}{\sqrt{2}})^2}\,\frac{\mathbf{i}}{\sqrt{2}}$$



B. 4+2 (P-65)

Find the electric field that Q is subjected to (created by $\pm\lambda$, a pair of infinite line charges $\perp$ to the paper). Compute the work done in moving Q from $(0,0)$ to $(x,0)$ in the xy-plane. The potential energy is trivial; the voltage all along the x-axis is zero, so $W = 0$.

$$\mathbf{E} = \frac{2\lambda k}{\sqrt{a^2+x^2}}\Big(\frac{x}{\sqrt{a^2+x^2}}\mathbf{i} + \frac{a}{\sqrt{a^2+x^2}}\mathbf{j}\Big)$$
$$+\frac{2\lambda k}{\sqrt{a^2+x^2}}\Big(-\frac{x}{\sqrt{a^2+x^2}}\mathbf{i} + \frac{a}{\sqrt{a^2+x^2}}\mathbf{j}\Big)$$

C. 6. (E-42)

In the region of space $y \geq 0$ there is an electric field with voltage function $V(x,y) = \frac{V_0}{\pi}\tan^{-1}\frac{y}{x}$. Find the equation of the field line through the point $(x,y) = (a,a)$.
equipotentials are straight lines through the origin; $V_1 = \frac{V_0}{\pi}\tan^{-1}\frac{y}{x}$, $\frac{y}{x} = \tan\frac{V_1\pi}{V_0}$, so electric field lines are circles centered on the origin (perpendicular to these equipots). Therefore the field line through (a,a) is $x^2+y^2 = a^2+a^2 = 2a^2$.

Problem 5. 18 (E-19,20,21,22,23,24)

Consider a $1000 Hz$ sound wave created by a human voice. You are standing $5\,m$ away, and its loudness is $50\,dB$. The speed of sound is $340\,\frac{m}{s}$, and its density is $1.4\frac{kg}{m^3}$.

A. 2 points. What is the amplitude with which the air molecules move, due to the passage of the wave?

$$A = \sqrt{\frac{1 \times 10^{-7}}{2\pi^2(1000)^2 \cdot 1.4 \cdot 340}} = 3.26 \times 10^{-9} m$$

B. 2 points. What is its intensity in Watts per square meter?

$$I = I_0 \cdot 10^{\frac{\beta}{10}} = 1 \times 10^{-7} \frac{W}{m^2}$$

C. 3 points. What is the pressure amplitude of the accompanying pressure wave? (compute the largest pressure deviation from atmospheric pressure caused by the wave)

Get v_{max} from part G

$$\Delta P = \gamma P_0 \frac{v_{max}}{c} = (1.4)(1.01 \times 10^5)(\frac{2.05 \times 10^{-5}}{340})$$

D. 3 points. If a second identical wave of twice the frequency, but of loudness $50\,dB$ is added onto this wave **in phase**, how many decibels will the combined wave be heard at by you?

$$\beta_{new} = \beta_{old} + 10 \log_{10} 2 = 53\,dB$$

E. 3 points. If a second wave of the same frequency, but of loudness $50\,dB$ is added onto this wave **in phase**, how many decibels will the combined wave be heard at by you?

$$\beta_{new} = \beta_{old} + 10 \log_{10} 4 = 56\,dB$$

F. 3 points. If you **quadruple** your distance from a $50\,dB$ sound source, what will be the new decibel level of the source?

$$\beta_{new} - 40\,dB = 10 \log_{10} \frac{\frac{\wp}{4\pi(4R)^2}}{\frac{\wp}{4\pi R^2}} = -10 \log_{10} 16 = -12.05 dB, \quad \beta_{new} = 38\,dB$$

G. 2 points. Find the maximal molecular speed of the air molecules in the original $50\,dB$ wave.

$$v_{max} = 2\pi f A = 2.05 \times 10^{-5} \frac{m}{s}$$