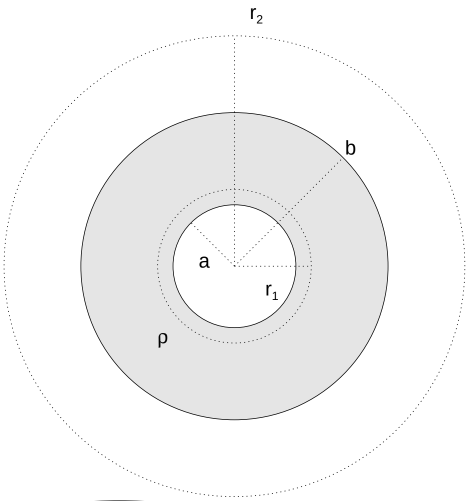
Physics 202 Exam 2, Name:________________________________



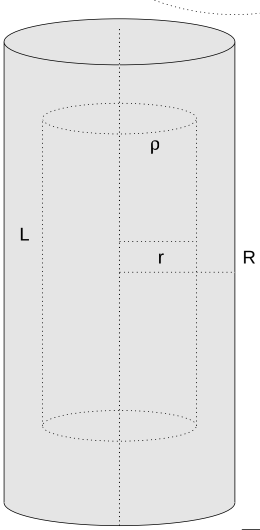
Problem 1a. $(4+4$, E-52)

A hollow ball of inner radius a and outer radius b is made of charged dust of density ρ. **Compute the electric field strength $|\mathbf{E}(r)|$ for $r < a$, and for $r > b$**

Solution

$$|E(r)|\,4\pi r^2 = \frac{0}{\epsilon_0}, \qquad |E(r)| = 0,\ r < a$$

$$|E(r)|\,4\pi r^2 = \frac{\rho\frac{4\pi}{3}(b^3-a^3)}{\epsilon_0}, \qquad |E(r)| = \frac{\rho(b^3-a^3)}{3\epsilon_0 r^2},\ r > b$$



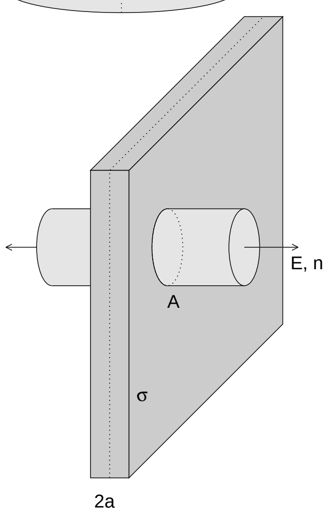
Problem 1b. (4+4 P-84)

Use Gauss' law to compute the electric field at a distance $r < R$ from the central axis of an infinitely long cylinder made of charged dust of density ρ, with radius R. Compute the field strength for some $r > R$.

Solution

$$|E(r)|\,2\pi r\ell = \frac{\rho\pi r^2\ell}{\epsilon_0}, \qquad |E(r)| = \frac{\rho r}{2\epsilon_0},\ r < R$$

$$|E(r)|\,2\pi r\ell = \frac{\rho\pi R^2\ell}{\epsilon_0}, \qquad |E(r)| = \frac{\rho R^2}{2\epsilon_0 r},\ r > R$$



Problem 1c. (4, P-89)

Compute the electric field strength at a point a distance x (measured perpendicularly) from the median plane (dotted) of an infinite planar slab of charged dust of density ρ and thickness $2a$, $x > a$.

Solution

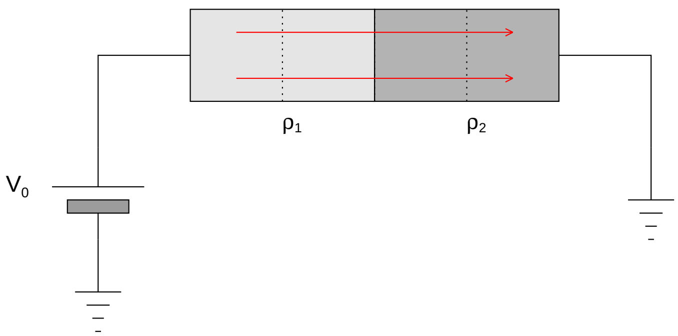
$$2A|E(x)| = \frac{2aA\rho}{\epsilon_0}, \quad |E(x)| = \frac{\rho a}{\epsilon_0}$$

Problem 2a. (2+2 E-116)

A vector potential for a time-varying magnetic field is $\mathbf{A} = 4.0\,T\,e^{-\frac{t}{10s}}(y\mathbf{i} - x\mathbf{j})$. **Find** both $\mathbf{B}$ and $\mathbf{E}$.

Solution

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} = 0.4\,\frac{T}{s}\,e^{-\frac{t}{10s}}(y\mathbf{i} - x\mathbf{j}), \qquad \mathbf{B} = \nabla \times \mathbf{A} = -8.0\,T\,e^{-\frac{t}{10s}}\,\mathbf{k}$$



Problem 2b. (4+4+4 P-129)

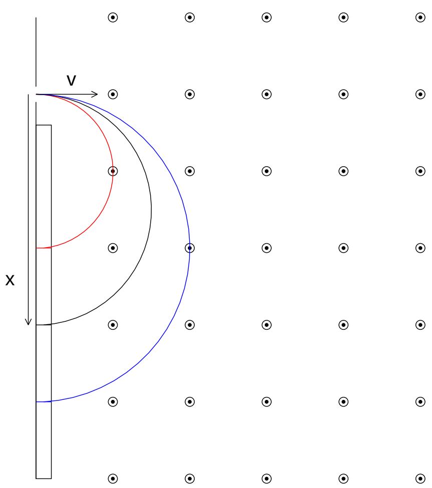
A wire of cross-sectional area $A = 2.0\times10^{-4}m^2$ and length $\ell = 0.5m$ is made of two equal length conductors with different resistivities $\rho_1 = 1.0\times10^{-9}\Omega\cdot m$ and $\rho_2 = 2.0\times10^{-9}\Omega\cdot m$. Find $\mathbf{J}$ and $\mathbf{E}$ inside of each half when it is connected to a ground and battery $V_0 = 10V$ as shown. Find the total resistance of the device.

Solution

$$\mathbf{J} = \frac{I}{A}\mathbf{i}, \qquad \mathbf{E}_1 = \rho_1\frac{I}{A}\mathbf{i}, \qquad \mathbf{E}_2 = \rho_2\frac{I}{A}\mathbf{i}$$

$$0 - V_0 = -\int_0^{\frac{\ell}{2}} \mathbf{E}_1\cdot d\mathbf{r} - \int_{\frac{\ell}{2}}^{\ell} \mathbf{E}_2\cdot d\mathbf{r} = -\frac{(\rho_1+\rho_2)I\ell}{2A}, \qquad I = 2.667\times10^6 A$$

$$|J_1| = |J_2| = 1.33\times10^{10}\frac{A}{m^2}, \qquad |E_1| = \rho_1 J_1 = 13.3\frac{N}{m}, \qquad |E_2| = \rho_2|J_2| = 26.6\frac{N}{m}$$

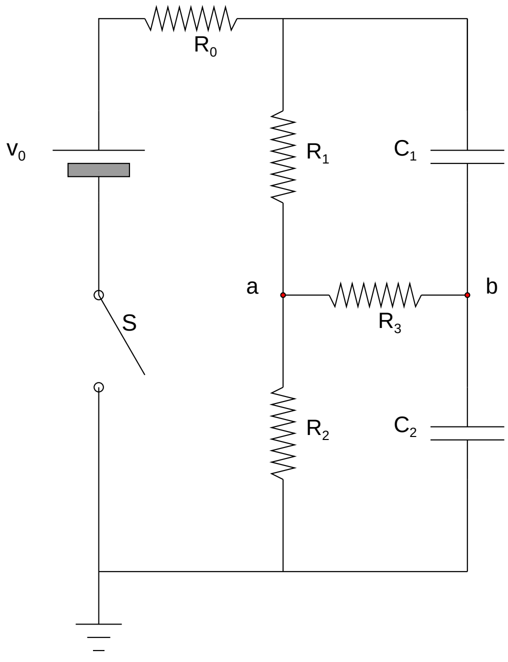


Problem 2c. (4, P-155)

The figure shows a mass spectrograph in which a charged particle of mass m and charge $q = 3.2\times10^{-19}C$ is accelerated from rest by passing it through a potential difference $V_0 = 5000V$ and injecting it into a magnetic field of strength $|B| = 0.8T$. The particle travels in a circular path in the chamber, because there is a uniform magnetic field (out of the paper) there. It strikes a photographic plate a distance $x = 0.5m$ from the entry point. Find its mass m.

Solution

$$\frac{1}{2}mv^2 = qV_0, \qquad \frac{mv^2}{\frac{x}{2}} = qvB, \qquad m = \frac{qB^2x^2}{8V_0} = 1.28\times10^{-24}kg$$



Problem 3a (6, p-133)

$V_0 = 100V$, $R_0 = 100\Omega$, $R_1 = 200\Omega$, $R_2 = 300\Omega$, $R_3 = 500\Omega$, $C_1 = 2\times 10^{-8}F$, $C_2 = 3\times 10^{-8}F$.
a. Instantly after closing the switch S at $t=0$, find the currents through the resistors I_0, I_1, I_3, I_2 **(all down or to the right)** and the charges on the capacitors q_1, q_2 **(top plates positive)**.

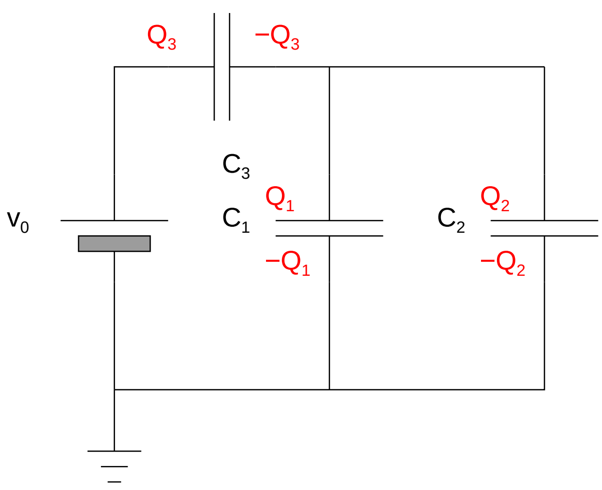
Solution. Instantly after closing S, $q_1 = q_2 = 0$ and all current routs through the capacitors; $I_1 = I_2 = I_3 = 0$, $\frac{V_0}{R_0} = I_0 = 1.0A$.

b.(6) Very long after closing the switch S at $t=0$, the currents will have reached steady (constant) values, as will the charges on the capacitors. Find these steady values.

Solution. The capacitors reach steady state, $I_0 = I_1 = I_2 = \frac{V_0}{R_0+R_1+R_2} = 0.1667A$, $I_3 = 0$,

$$0 = \frac{Q_1}{C_1} + R_3\cdot 0 - R_1 I_1, \qquad Q_1 = 6.67\times 10^{-7}C$$

$$0 = \frac{Q_2}{C_2} + R_3\cdot 0 - R_2 I_2, \qquad Q_1 = 1.5\times 10^{-6}C$$



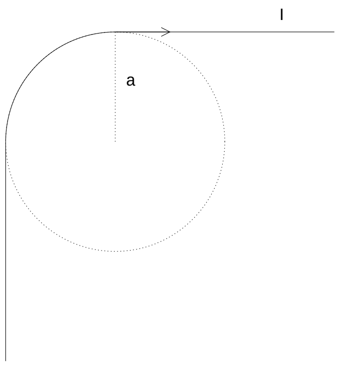
Problem 3b (2+2+2+2, P-133)

Find Q_1, Q_2, Q_3 along with the total work done by the battery in charging the entire system. $C_1 = 1\times 10^{-9}F$, $C_2 = 2\times 10^{-9}F$, $C_3 = 3\times 10^{-9}F$, $V_0 = 10V$.
Solution

$$Q_3 = Q_1 + Q_2, \quad V_0 = \frac{Q_3}{C_3} + \frac{Q_1}{C_1}, \quad Q_2 = Q_1\frac{C_2}{C_1}$$

$$Q_1 = \frac{C_1C_3}{C_1+C_2+C_3}V_0, \qquad Q_2 = \frac{C_2C_3}{C_1+C_2+C_3}V_0$$

$$Q_3 = \frac{(C_1+C_2)C_3}{C_1+C_2+C_3}V_0, \qquad W_B = V_0Q_3$$

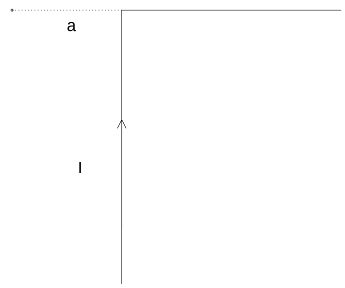


Problem 4a (5)

Use the Biot-Savart law to find the magnetic field at the center of the circle shown. The straight segments are semi-infinite, carrying I.

Solution

$$\mathbf{B} = \frac{2}{2}\frac{\mu_0 I}{2\pi a} + \frac{1}{4}\frac{\mu_0 I}{2a}\,(in)$$

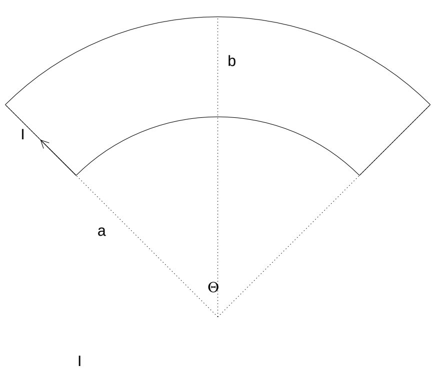


Problem 4b (5)

Use the Biot-Savart law to find the magnetic field at the point illustrated. The straight segments are semi-infinite, carrying I.

Solution

$$\mathbf{B} = \frac{1}{2}\frac{\mu_0 I}{2\pi a}\,(out)$$

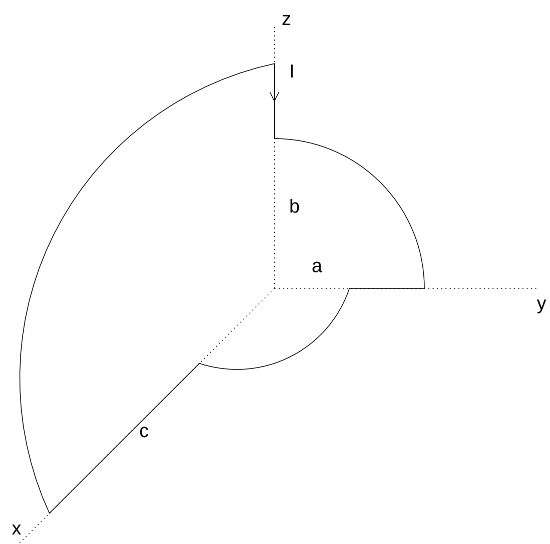


Problem 4c (5)

Use the Biot-Savart law to find the magnetic field at the center of the circles that the two arcs are sections of.

Solution

$$\mathbf{B} = \left(\frac{\mu_0 I \Theta}{4\pi a} - \frac{\mu_0 I \Theta}{4\pi b}\right)(out)$$

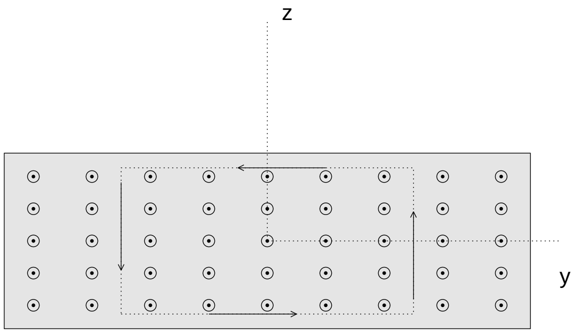


Problem 4d (5)

Use the Biot-Savart law to find the magnetic field at the origin. The arcs are circular

Solution

$$\mathbf{B} = \left(\frac{\mu_0 I}{8a}(-\mathbf{k}) + \frac{\mu_0 I}{8b}(-\mathbf{i}) + \frac{\mu_0 I}{8c}(-\mathbf{y})\right)$$

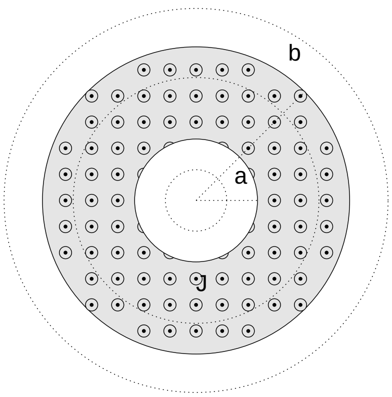


Problem 5a (4)

Compute the magnetic field at a distance z above and below the median plane (dotted y-axis lies on it) within a planar slab of thickness $2D$, $D > z$ carrying a uniformly distributed current density $\mathbf{J} = J\,\mathbf{i}$ (in $\frac{Amp}{m^2}$) out of the paper. The figure shows the plane (rather part of it) in cross-section.

Solution

$$\oint_C \mathbf{B}\cdot d\ell = 2|B|\ell = \mu_0(2z\ell)J, \qquad |B| = \mu_0 J z$$



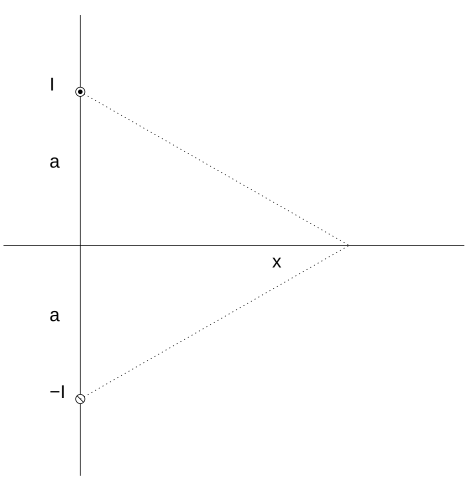
Problem 5b (4+4+4)

The left figure shows a long conducting hollow tube carrying a uniform current I out of the page, distributed evenly over the cross sectional conducting area $\pi(b^2 - a^2)$. Determine from Ampere's law $|\mathbf{B}(\mathbf{r})|$ for $r < a$, $a < r < b$, and $r > b$. I have drawn suitable Amperian loops for you.

Solution

$$\oint_C \mathbf{B}\cdot d\ell = |B|2\pi r = \mu_0\cdot 0 \qquad |B| = 0 \quad r < a, \qquad \oint_C \mathbf{B}\cdot d\ell = |B|2\pi r = \mu_0 I \qquad |B| = \frac{\mu_0 I}{2\pi r} \quad r > b,$$

$$\oint_C \mathbf{B}\cdot d\ell = |B|2\pi r = \mu_0 I\frac{\pi(r^2-a^2)}{\pi(b^2-a^2)} \qquad |B| = \frac{\mu_0 I}{2\pi r}\frac{\pi(r^2-a^2)}{\pi(b^2-a^2)} \quad a < r < b$$



Problem 5c (4)

Find the magnetic field at a point $\mathbf{r} = x\mathbf{i}$ created by two wires parallel to the z-axis carrying currents I in opposite directions (top one is out, bottom is into the paper).

Solution

$$\mathbf{B} = 2\frac{\mu_0 I}{2\pi\sqrt{x^2+a^2}}\frac{a}{\sqrt{x^2+a^2}}\,\mathbf{i}$$