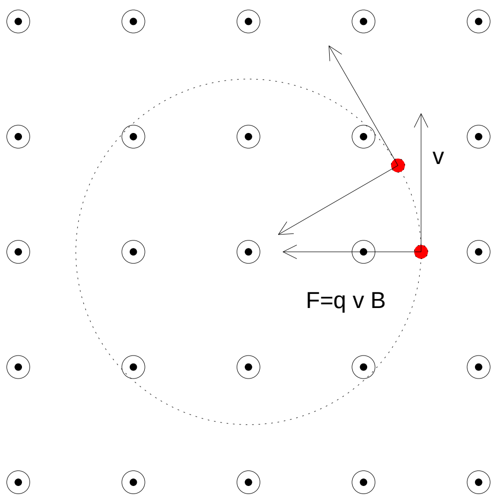
8 Electron charge-to-mass ratio

The purpose is to use the deflection of electrons in a magnetic field to obtain the ratio of their charge to mass ($\frac{q}{m}$).

The apparatus is an integrated e/m experimental apparatus and the Heathkit $IP-32$ power supply. The e/m apparatus has a Helmholtz coil, and electron gun, and the Heathkit supply has outputs to supply current to the coils, to supply the accelerating voltage for the electron gun and to power the gun heater element that creates free electrons by thermionic emission. In addition you will need a DC ammeter and a DC voltmeter.

8.1 Electron orbits in a magnetic field



A beam of electrons can be produced by applying an accelerating voltage to pull electrons away from a heater filament that is ejecting electrons via thermionic emission. If the accelerating voltage is V_0, the speed v of the electrons is related to the accelerating voltage

$$\frac{1}{2} m\, v^2 = q\, V_0, \qquad v = \sqrt{\frac{2\,|q|\,|V_0|}{m}}$$

The beam enters a uniform magnetic field $|\vec{B}|$, which acts to bend the beam in a circular arc of radius r.

For circular motion we have

$$m\,|\vec{a}_c| = \frac{m\vec{v}^2}{r} = |q\,\vec{v} \times \vec{B}|$$

which simplifies to

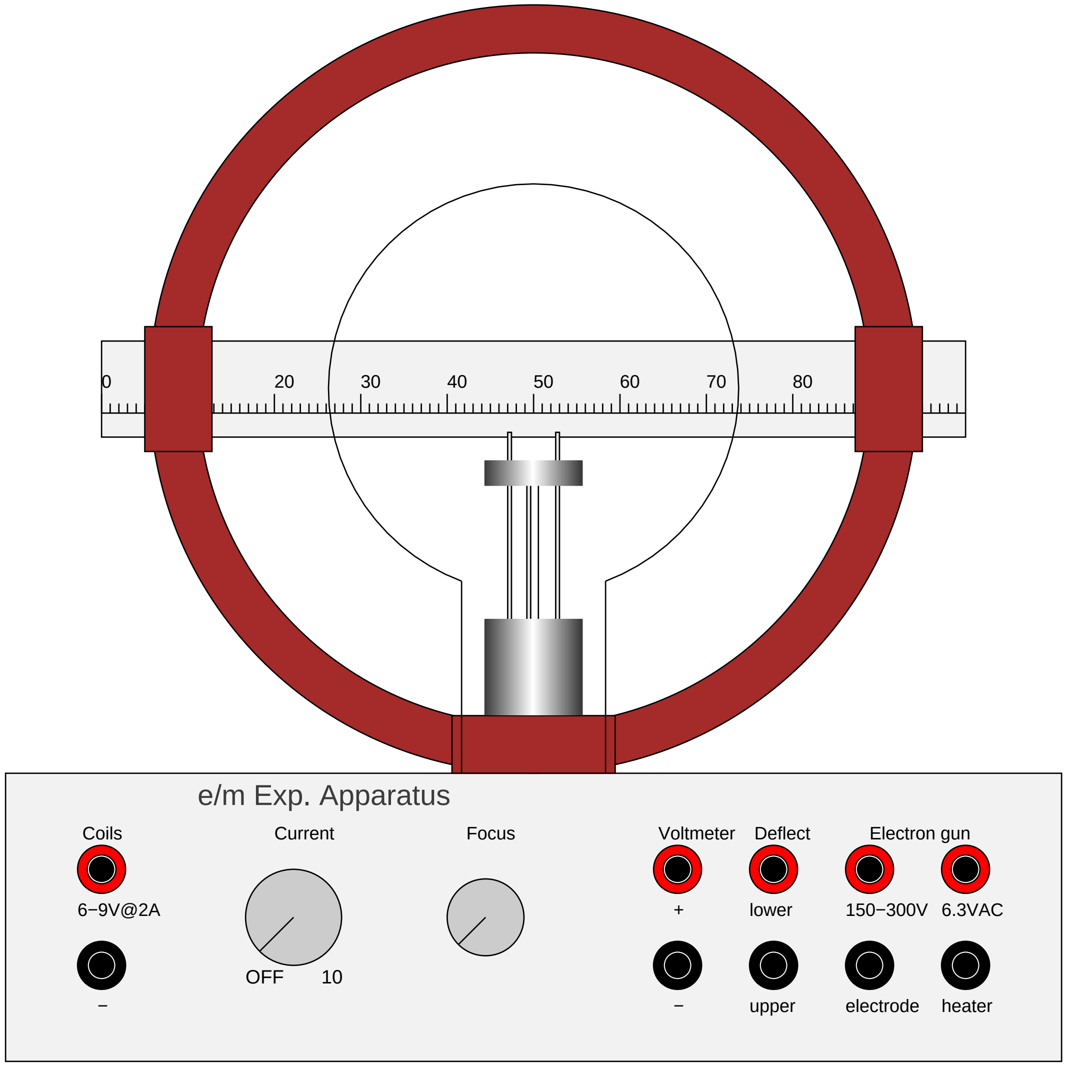
$$m|\vec{v}| = |q||\vec{B}|r$$

The charge-to-mass ratio $\frac{q}{m}$ can be calculated if one knows the radius of the circular path, the field strength, and the accelerating voltage

$$\frac{|q|}{m} = \frac{|\vec{v}|}{|\vec{B}|r} = \frac{1}{|\vec{B}|\,r}\sqrt{\frac{2\,|q|\,|V_0|}{m}} = \frac{2|V_0|}{(r\,|\vec{B}|)^2}$$

8.2 Helmholtz coils

In order to provide the roughly uniform magnetic field needed in this experiment, we will use a set of **Helmholtz** coils such as those shown in the apparatus pictured below



Two coils of N turns each carry a current I, and have separation equal to their radius R. The field at a point z, with the origin half-way between the coils, is

$$B_z(z) = \frac{N\mu_0 I R^2}{2\left((\frac{R}{2} - z)^2 + R^2\right)^{\frac{3}{2}}} + \frac{N\mu_0 I R^2}{2\left((\frac{R}{2} + z)^2 + R^2\right)^{\frac{3}{2}}}$$

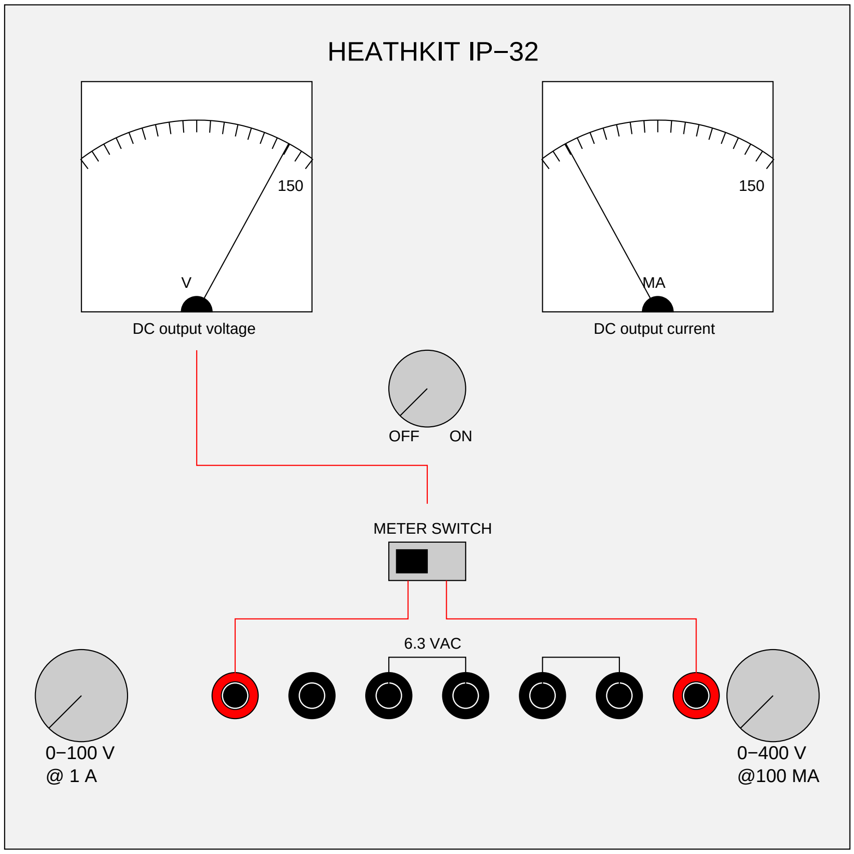
At the origin this simplifies to

$$B_z = \frac{N\mu_0 I}{R(\frac{5}{4})^{\frac{3}{2}}} = \frac{8N\mu_0 I}{\sqrt{125}R}$$

with

$$\mu_0 = 4\pi \times 10^{-7} \frac{Wb}{Am}$$

This field turns out to be surprisingly uniform, small variations in z or from the central axis produce no noticeable change in the strength of the field.



8.3 Procedure

Connect the circuits shown below in the figure. See that the meters are placed well away from the Helmholtz coils. Have the instructor check the circuit before proceeding.

Allow the cathode to heat for at least two minutes. Apply $100V$ potential to the plates and you will see a blue stream of electrons forming the beam. Adjust the plate voltage and the coil currents until the electron beam bends into a complete semi-circle with a radius that can be read from the mirrored scale. Record plate potential, coil current, and the two radii of the beam as measured from each side of the scale. Repeat the observations with somewhat different potential and current settings to produce a variety of beam radii. For each set of measurements compute $\frac{q}{m}$. Finally compute an average charge to mass ratio, a standard deviation, and a percent error as compared to the exact value.

8.4 Pre-lab questions

1. Show that the field at the center of the Helmholtz coil is

$$B_z = \frac{8N\mu_0 I}{\sqrt{125}R}$$

2. Show that the field at a point z from the center of the Helmholtz coils is (along the axis)

$$B_z(z) = \frac{N\mu_0 I R^2}{2\Big((\frac{R}{2} - z)^2 + R^2\Big)^{\frac{3}{2}}} + \frac{N\mu_0 I R^2}{2\Big((\frac{R}{2} + z)^2 + R^2\Big)^{\frac{3}{2}}}$$

and show that the derivative of this for $z = 0$ vanishes. That means that at the center of the coils, the field is extremely uniform (no spatial variations).

3. If electrons are ejected by thermionic emission from the heater with nearly zero velocity, how fast are they traveling after being accelerated by the potential difference $V_0 = 100\,V$?

4. The beam of electrons in this experiment will be visible because the path will glow a faint blue. Why does this happen? Are the electrons emitting blue light as the beam bends? Would this happen if you broke the glass of the apparatus tube?

5. Show that for a particle of fixed $\frac{q}{m}$, doubling the velocity will double the radius of the circular path followed by the particle in a uniform magnetic field.

8.5 Lab report

Electron charge-to-mass ratio

Experimenter 1 ______________________ **Experimenter 2** ______________________

Experimenter 3 ______________________ **Experimenter 4** ______________________

The Helmholtz coils

Mean radius $R =$ ______________________ (m) Turn number $N =$ ______________________

The electron paths

Electron paths					
Trial	Coil current I (Amp)	V_0 (V)	Mean path radius r (m)	$\|\vec{B}\|(T)$	$\frac{\|q\|}{m}$ (C/kg)
1					
2					
3					
4					
5					
6					

Average $\frac{|q|}{m} =$ ______________________ $\sigma_{\frac{|q|}{m}} =$ ______________________

$\left(\frac{|q|}{m}\right)_{theor} =$ ______________________ $\%error =$ ______________________