5 Measurements

The purpose of this experiment is to learn about basic measurement techniques, types of errors associated with measurement, and how errors propagate through calculations.

The equipment used is the Vernier caliper, micrometer, and triple beam balance or alternatively the electronic balance. Data is taken for wooden blocks and dowels and metal cylinders.

If a quantity, for example the density of a certain material, is to be experimentally determined, it can either be measured directly (with a "density meter" if you can find one), or indirectly using related quantities that are also measured. For example if a cylinder is found to have a height h, diameter D, and mass m, all measured quantities, then the density can be computed from

$$\rho = \frac{m}{V} = \frac{4m}{\pi D^2 \, h}$$

The problem is that the radius and height may not be uniform, since the cylinder may not be perfect. In fact, the purpose of this lab is to show you that you can still get good results with imperfect data, provided that one understands the concepts of statistics and error propagation. For this reason the cylinders or blocks have been deliberately made non-uniform.

If one makes N measurements of the diameter $D_1, D_2, D_3, \cdots, D_N$, at various positions on the cylinder, and N height measurements $L_1, L_2, L_3, \cdots, L_N$, also made at various positions, a **mean** or average can be estimated

$$\bar{D} = \frac{\sum_{n=1}^{N} D_n}{N}, \qquad \bar{L} = \frac{\sum_{n=1}^{N} L_n}{N}$$

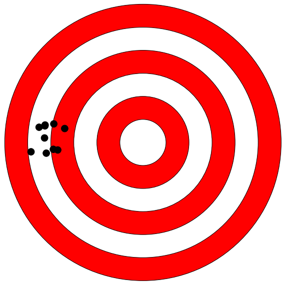
If we do the same for the mass, a good estimate of the density is then

$$\bar{\rho} = \frac{4\bar{m}}{\pi \bar{D}^2 \, \bar{L}}$$

How good is the estimate? Limits to the precision can be found in the following way. The standard deviations of the diameter and length measurements are estimated as

$$\sigma_D = \sqrt{\frac{1}{N-1}\sum_{n=1}^{N}(D_n - \bar{D})^2}, \qquad \sigma_L = \sqrt{\frac{1}{N-1}\sum_{n=1}^{N}(L_n - \bar{L})^2}$$

If these quantities are small, we say that the measurements were **precise**, meaning all pretty much the same. Precise does not mean accurate though.



Precise shooting　　　Accurate shooting

Is the shooter suffering from a systematic error, or random?

To get you started, I will give you the requisite derivatives for the error propagation on the cylinder;

$$\rho_D(D,L,m) = -2\frac{4m}{\pi D^3 L}, \quad \rho_L(D,L,m) = -\frac{4m}{\pi D^2 L^2}, \quad \rho_m(D,L,m) = \frac{4}{\pi D^2 L}$$

and now apply the rules for computing the standard deviation in ρ;

$$\sigma_\rho^2 = \Big(-2\frac{4\bar{m}}{\pi \bar{D}^3 \bar{L}}\,\sigma_D\Big)^2 + \Big(-\frac{4\bar{m}}{\pi \bar{D}^2 \bar{L}^2}\,\sigma_L\Big)^2 + \Big(\frac{4}{\pi \bar{D}^2 \bar{L}}\,\sigma_m\Big)^2$$

and this value is what we accept as the **propagated error or uncertainty** in the computed density. We say that

$$\rho_{measured} = \bar{\rho} \pm \sigma_\rho, \qquad \sigma_\rho = \sqrt{\Big(-2\frac{4\bar{m}}{\pi \bar{D}^3 \bar{L}}\,\sigma_D\Big)^2 + \Big(-\frac{4\bar{m}}{\pi \bar{D}^2 \bar{L}^2}\,\sigma_L\Big)^2 + \Big(\frac{4}{\pi \bar{D}^2 \bar{L}}\,\sigma_m\Big)^2}$$

The measurements resulted in an accurate determination of the density of the material of the cylinder if the actual density ρ_{true} is within the error bars of the experiment

$$\bar{\rho} - \sigma_\rho \le \rho_{true} \le \bar{\rho} + \sigma_\rho$$

If we had used a rectangular block of material, of dimensions L, H, W, then our formulas would read

$$\bar{\rho} = \frac{\bar{m}}{\bar{L}\,\bar{W}\,\bar{H}}$$

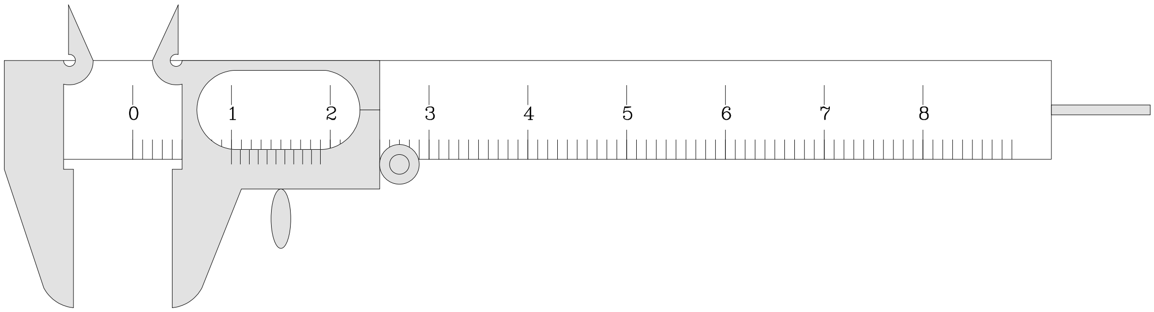
and

$$\sigma_\rho^2 = \Big(-\frac{\bar{m}}{\bar{L}^2 \bar{W} \bar{H}}\,\sigma_L\Big)^2 + \Big(-\frac{\bar{m}}{\bar{L} \bar{W}^2 \bar{H}}\,\sigma_W\Big)^2 + \Big(-\frac{\bar{m}}{\bar{L} \bar{W} \bar{H}^2}\,\sigma_H\Big)^2 + \Big(\frac{1}{\bar{L} \bar{W} \bar{H}}\,\sigma_m\Big)^2$$

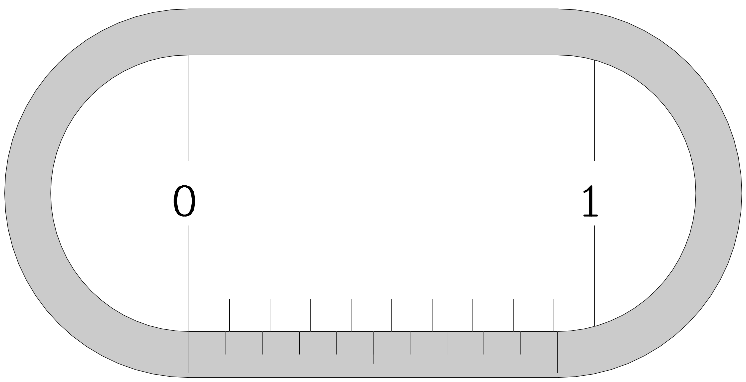
5.1 Experimental procedure

Choose a block of wood, and a metal cylinder as your experimental subjects. Use the triple beam balance or electronic balance to measure its mass. The balances are very much more accurate than our other measurements, so **we only take one measurement of the mass.** The standard deviation of a single point of data is zero.

Use the vernier caliper to measure the block's length, width and height, each five times, at different points on the block.

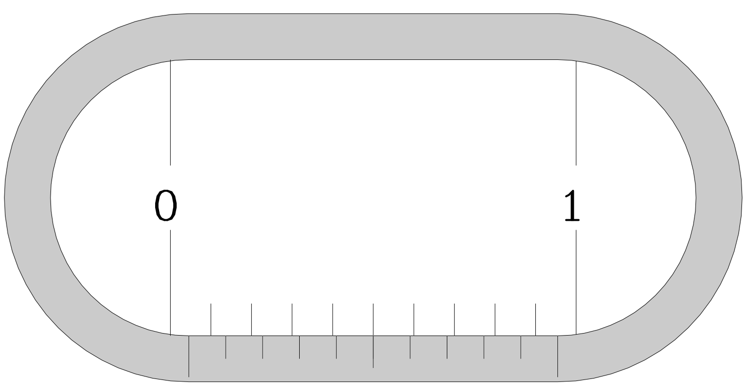


The vernier caliper is a fairly precise and easy to use tool for measuring small lengths. There is a gross scale and a fine scale. The diagram below will illustrate how to use the two scales in a measurement process.



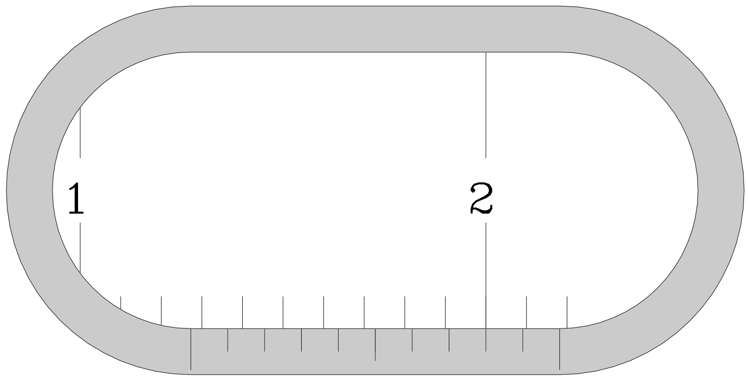
Caliper reads 0.0

In the figure below, the zero of the outer scale lies between 0 and 0.1, and so the length x being measured is $0.0 < x < 0.1$. We now examine which outer scale division lines up most closely with an inner scale division to establish the next decimal place. The 5^{th} division on the outer scale is closest to lining up with an inner scale division, and so $x \approx 0.05$.



Caliper reads 0.05

The outer fine scale in the figure below indicates that the measured value is between 1.2 and 1.3, by examining which of its rulings (the eighth) lines up best with the rulings on the gross scale, we establish that the caliper reads 1.28.



Caliper reads 1.28

For each measurement reset the caliper to some standard value and measure different spots on the block each time. Repeat all of these measurements on a cylinder, making five each of diameter and length measurements. If you are using a digital electronic scale, just make one mass measurement for each object.

5.2 Data analysis

For each of the quantities measured, compute the mean value of the measurement and the standard deviation, and record the data in the table provided.
For example

$$\bar{L} = < L > = \frac{L_1 + L_2 + L_3 + L_4 + L_5}{5}$$

$$\sigma_L^2 = \frac{(L_1 - \bar{L})^2 + (L_2 - \bar{L})^2 + (L_3 - \bar{L})^2 + (L_4 - \bar{L})^2 + (L_5 - \bar{L})^2}{4}$$

Compute the density of the material of the block and cylinder, along with it's standard deviation. Get the true values from your lab instructor, the CRC or from the appendix, and determine whether or not your measurements produced reliable results. State in your conclusions whether or not the true density is within the error bars of your computed averages, and give the **percent error** if asked for it;

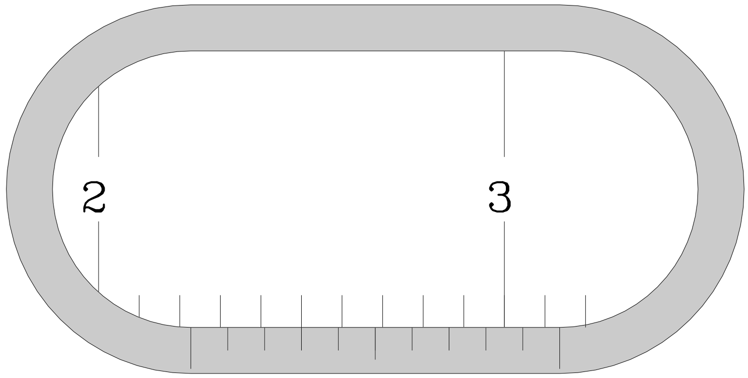
$$\%error = \frac{|\rho_{true} - \bar{\rho}|}{\rho_{true}} \times 100\% \qquad \text{or alternatively} \qquad \%error = \frac{|\rho_{true} - \bar{\rho}|}{|\rho_{true} + \bar{\rho}|} \times 200\%$$

If a digital scale is used to measure the mass of the various experimental subjects, repeated measurements will result in the same mass each time, and so we use $\sigma_m = 0$ for the mass measurements.

5.3 Pre-lab questions

1. Suppose that you used a digital scale to measure the mass of a block five times, the first measurement being m_1, the last m_5, and got five results $m_1 < m_2 < m_3 < m_4 < m_5$. Explain how such a result could occur. What type of error (random or systematic) has been introduced into the experiment?

2. What does the caliper below read?



3. Suppose that the density of the material from which a block is made is

$$\rho = \frac{m}{LWH}$$

and that you have made ten measurements each of L, W and H. The results are that $\bar{H} < \bar{W} < \bar{L}$, but $\sigma_L = \sigma_H = \sigma_W$. Which quantity, length, width or height dominates or contributes the most to σ_ρ?
There is a very significant generalization to be made here regarding which of your data will dominate the precision of your measurements.

4. Write down the appropriate propagation of errors formula for each of the following equations.
a. $f = \frac{mv^2}{r}$

b. $V = L^3$

Fill in all of your data and the **outcomes** of your computations on the following data sheet and put the answers to the above questions on the spaces alloted for them . Hand in **only** this data sheet as your lab report. **This must be handed in before you leave the lab**.

Web-based lab report If the web-interface is available, perform your analysis and generate your lab report using the browser (Netscape). You will get some feedback during the process that may answer some of your questions about the experiment and the analysis. The program will check your data and computations for consistency and obvious errors.

5.4 Lab report

The next two pages constitute a lab report, to be completed and filled out **in class** and handed in at the end of lab. This is available online, and the CGI interface can partially debug your data and analysis.

Measurements

Experimenter 1 ____________________ **Experimenter 2** ____________________

Experimenter 3 ____________________ **Experimenter 4** ____________________

$m_{block}\,(grams)$ ____________________ $m_{cylinder}\,(grams)$ ____________________

Data for Block			
Trial i	L_i (cm)	W_i (cm)	H_i (cm)
1			
2			
3			
4			
5			
Averages			
Std. Dev.			
$\rho_{block} \pm \sigma_\rho$			

Data for Cylinder		
Trial i	L_i (cm)	D_i (cm)
1		
2		
3		
4		
5		
Averages		
Std. Dev.		
$\rho_{cylinder} \pm \sigma_\rho$		