202 final exam sample

Ray Optics Expressions

$$\frac{1}{O}+\frac{1}{I}=\frac{1}{f},\ f=\frac{R}{2},\qquad \frac{1}{f}=(\frac{n_\ell-n_{med}}{n_{med}})(\frac{1}{R_1}+\frac{1}{R_2}),\quad v=\frac{c}{n}$$

$$m=-\frac{I}{O},\qquad \theta_{inc}=\theta_{ref}$$

Transmission/Reflection Expressions

$$E_R=E_0\frac{-n_1\cos\theta_T+n_2\cos\theta}{n_1\cos\theta_T+n_2\cos\theta},\qquad E_T=E_0\frac{2n_1}{n_1+n_2},\qquad I_{trans}=I_0\cos^2\theta,\quad E_R=E_0\frac{n_1\cos\theta-n_2\cos\theta_T}{n_1\cos\theta+n_2\cos\theta_T}$$

$$\tan\theta_B=\frac{n_2}{n_1},\qquad n_1\sin\theta_I=n_2\sin\theta_T,\qquad n_1\sin\theta_{crit}=n_2\sin 90^o\quad f_{inc}=f_{trans}=f_{ref}$$

General Light Expressions

$$\vec{S}=\vec{Y}=I\hat{n}=\frac{1}{\mu_0}\vec{E}\times\vec{B},\qquad \frac{dU_E}{dVol}=\frac{1}{2}\epsilon_0\vec{E}^2,\qquad \frac{dU_B}{dVol}=\frac{1}{2\mu_0}\vec{B}^2\qquad \bar{I}=\frac{\bar{\wp}}{Area},\quad |\vec{F}|=|\frac{d\vec{p}}{dt}|$$

$$p=\frac{\bar{I}LA}{c^2},\qquad v=\frac{c}{n}=\frac{\omega}{k}=f\lambda,\qquad |B|=\frac{|E|}{v},\qquad I=I_0\cos^2\theta,\quad r_+-r_-=m\lambda$$

$$\vec{E}=E_0\,\hat{p}\,\sin(k\hat{n}\cdot\vec{r}-\omega t),\qquad f\lambda=\frac{\omega}{k}=v,\qquad k=\frac{2\pi}{\lambda},\qquad \bar{I}=\frac{1}{2\mu_0 v}E_0^2,\qquad U=\sqrt{m^2c^4+p^2c^2}$$

$$J_d=\epsilon_0\frac{d}{dt}\vec{E},\qquad \oint_C\vec{B}\cdot d\vec{\ell}=\mu_0 I_{cond}+\mu_0\epsilon_0\frac{d}{dt}\int\vec{E}\cdot\hat{n}dA,\qquad \oint_C\vec{E}\cdot d\vec{\ell}=-\frac{d}{dt}\int\vec{B}\cdot\hat{n}dA$$

Constants

$$c=3.0\times10^8\frac{m}{s},\qquad \mu_0=4\pi\times10^{-7}\frac{T\cdot m}{A},\qquad \omega=2\pi f=\frac{2\pi}{T}$$

$$c^2=\frac{1}{\mu_0\epsilon_0},\qquad \frac{1}{4\pi\epsilon_0}=9\times10^9\frac{Nm^2}{C^2},\qquad n_{air}=1.0003,\qquad n_{H_2O}=1.333$$

Interference/Diffraction Expressions

$$\theta=\frac{1.22\lambda}{d},\qquad 2nd=m\lambda,\qquad 2nd=(m+\frac{1}{2})\lambda,\qquad I=I_0\cos^2(\frac{\pi d\sin\theta}{\lambda})$$

$$d\sin\theta=m\lambda,\qquad a\sin\theta=m\lambda,\qquad I(\theta)=I_0(\frac{\sin\alpha}{\alpha})^2,\qquad \alpha=\frac{\pi a\sin\theta}{\lambda}$$

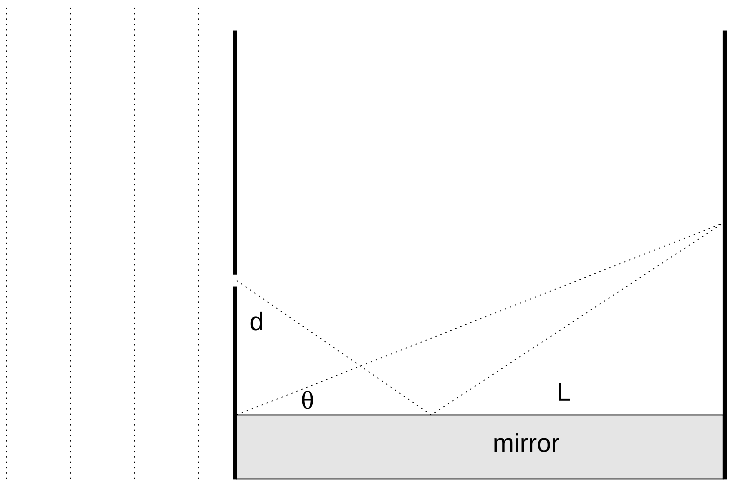
AC-Circuits

$$Z=\sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}=\frac{R}{\cos\phi},\qquad \omega=\frac{2\pi}{T},\qquad \tan\phi=\frac{\omega L-\frac{1}{\omega C}}{R}$$

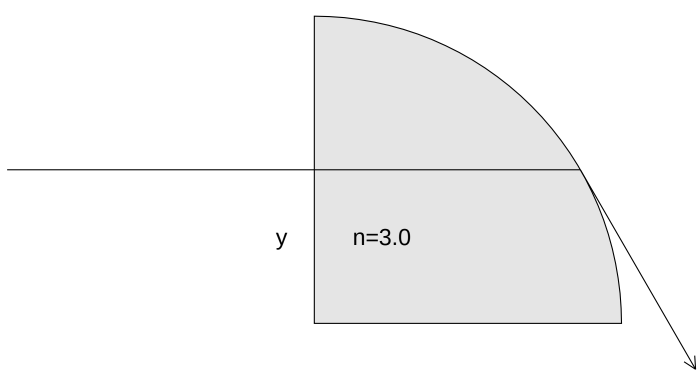
$$I=\frac{\mathcal{E}_0}{Z}\sin(\omega t-\phi),\qquad \omega_R=\frac{1}{\sqrt{LC}},\qquad V_R=IR,\qquad V_L=L\frac{dI}{dt},\qquad V_C=\frac{Q}{C}=\frac{\int I\,dt}{C}$$

$$(V_R)_{max}=I_{max}R,\qquad (V_L)_{max}=I_{max}\omega L,\qquad (V_C)_{max}=I_{max}\frac{1}{\omega C}$$

Problem 1. 14 points **Name; ______________________________**



A. 5+2 points Light of wavelength $\lambda = 400\,nm$ **in air** is incident on a one-narrow slit screen with $d = 0.5\,mm$ as illustrate. To the right of the aperture screen there is a horizontal mirror and a screen $L = 5.0m$ away upon which an interference pattern will be formed.
Find the angle θ at which the $m = 1$ bright interference fringe will be formed. Determine the y value for it; how high above (or below) the optical axis is this fringe located.



B. 7 points Compute the height y above the bottom face at which a ray of light parallel to the bottom surface of a quarter-cylinder prism must enter the prism of radius $R = 5.0\,cm$ in order to exit at the angle for total internal reflection.

Problem 2. 14 points

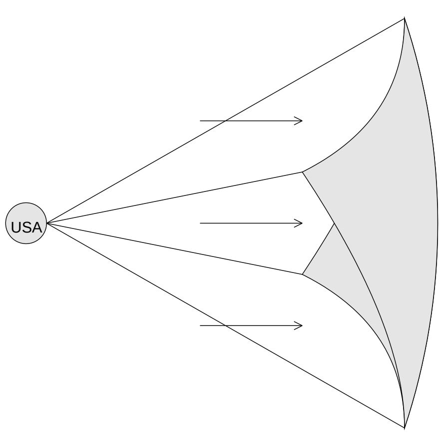
Consider the electric field of a light wave

$$\vec{E} = (100\frac{N}{C})\,\hat{k}\ \sin\Big(3.0\times 10^{7}\frac{1}{m}\,(0.5\hat{\imath} + 0.866\hat{\jmath})\cdot\vec{r} - 3.0\times 10^{15}\frac{rad}{s}t\Big)$$

A. 2+1
Determine wavelength and frequency of the light. **Full credit for numerical answers.**

B. 3 points
Determine the intensity of the light. **Full credit for numerical answers.**

C. 3 points
Write down a similar formula for the magnetic field $\vec{B}$ of the light. Give explicit numerical values for its magnitude and direction. **Full credit for numerical answers.**



D. 4 points A space probe is propelled by a solar sail; a large reflective sheet of material of area $A = 10,000\,m^2$ that reflects solar radiation of intensity $\bar{I}_{sol} = 1200\,\frac{W}{m^2}$. The probe has a mass of $m = 1000\,kg$, determine the acceleration that it gets when radiation is normally incident on the sail.

Problem 3. 16 points

A. 4 points An object and a viewing screen are place 100 cm apart. There are two positions **between** the object and screen where a lens can be placed such that there will be a **real** image cast on the screen. If the magnitude of the lens focal length is $|f| = 16\,cm$, find both positions and the required sign of f.

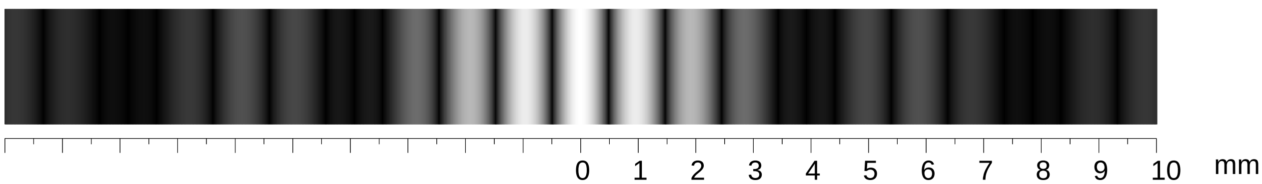
C. 4 points A lens-shaped bubble, underwater ($n_{water} = 1.333$) filled with air in the shape of a bi-concave lens of radii-magnitudes $|R_1| = |R_2| = 50\,cm$ is used to view a fish 40 cm away from the bubble. Find the focal length of the bubble, the image distance and magnification of the fish.

D. 4 points An object is placed 48 cm away from a biconcave lens of radii $|R_1| = 40\,cm$, $|R_2| = 60\,cm$. The lens has index of refraction $n = 2$ and is being used in air. Draw a ray diagram illustrating image formation. **Neatness counts, straight lines, proper scale and proportion, and the position of the eye.**

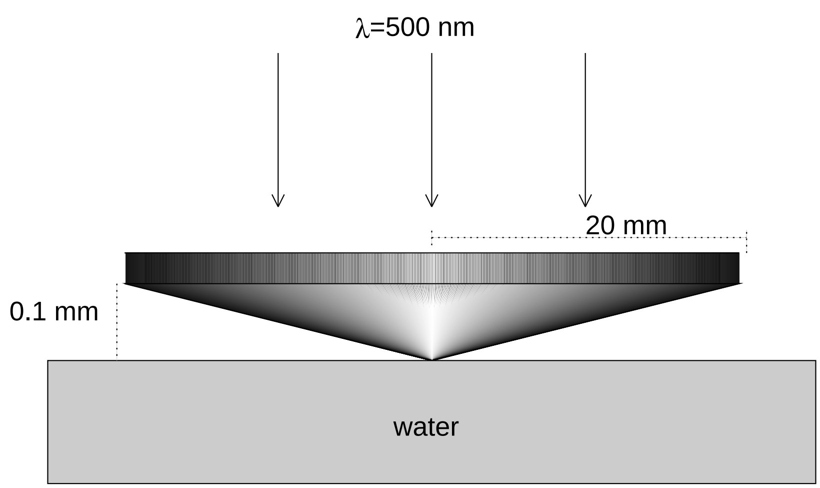
E. 4 points A mirror creates an image of some object at $\mathcal{O} = 25\,cm$ of magnification $m = 0.25$. Find the image distance I, focal distance f, and tell if you are looking into the concave side ($R > 0$) or convex ($R < 0$).

Problem 4. 14 points

A. 3+3 The figure shows a diffraction-interference pattern cast on a screen $L = 4.0\,m$ away from the slits that are the sources of the light. The horizontal scale is in units of millimeters, and the figure represents a diffraction pattern cast on the wall with a millimeter scale below it. Determine both the aperture width a and separation d if the pattern was created with light of $\lambda = 600\,nm = 6.0 \times 10^{-7}\,m$.

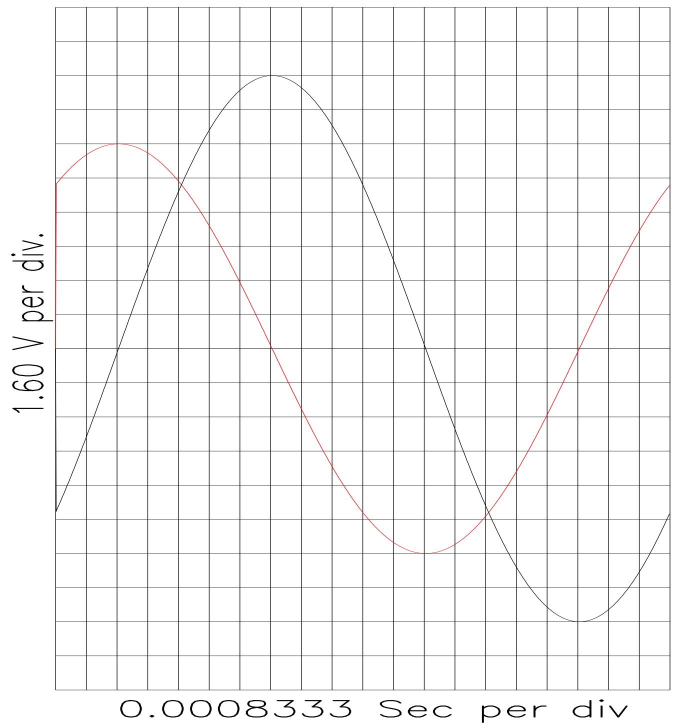


B. 4+4 A chip of glass of index of refraction $n = 2.2$ in the shape of a solid cone rests point-first in a puddle of water of index of refraction $n = 1.333$, of nearly zero depth. The flat, circular face of the chip faces upwards. It is illuminated above by light of the wavelength indicated. Looking down on it you see concentric rings of bright and dark interference fringes.



How many concentric dark rings will you actually see? What is the radius of the 10^{th} dark ring?

Problem 5. 18 points



Consider the AC circuit signal to the left, voltage across resistor (large signal), and voltage across a second device. (Don't tell U.J. that I said so, but its an inductor).

Given that $R = 100\Omega$, find ϕ, Z, ω, C, or L, and the maximum value of the current (current amplitude). **Numerical answers! 4+4+4+4+2 points**. Circle your answers.

Problem 6. 14 points

A beam of light with electric field

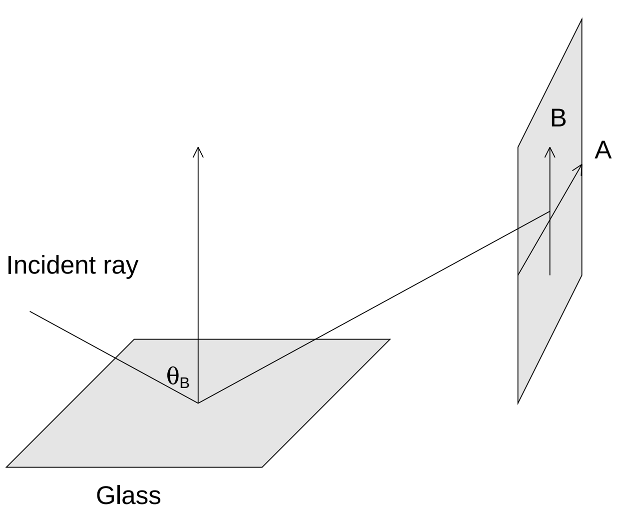
$$\vec{E} = 20\frac{N}{C}\hat{\imath}\,\sin\left(3.0\times10^{7}\frac{1}{m}\,z - 6.0\times10^{15}\frac{1}{s}\,t\right)$$

is normally incident on a stack of polarizing filters. The first polarizer has its axis in the $\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}$ direction, the second has axis $\hat{\jmath}$, and the third $\frac{\hat{\imath}+\sqrt{3}\,\hat{\jmath}}{2}$

A. 4 points What is the electric field of the light that is transmitted through the first polarizer?

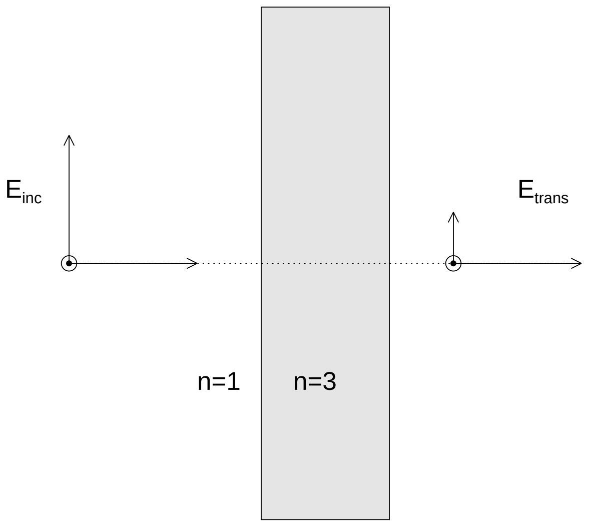
B. 3 points What is the electric field of the light that is transmitted through the second polarizer?

C. 3 points What intensity of light is transmitted through the entire stack?



D. 4 points Light is incident at Brewsters angle on a horizontal sheet of glass. The vertical polarizer lets no reflected light pass if it is in one of the two orientations, **A** or **B**. Which is it?

Problem 7. 10 points



A. 4 points Light is incident normally from air onto a sheet of high-quality glass of index of refraction equal to 3.0. The incident light is unpolarized and has intensity $I_{inc} = 100\frac{W}{m^2}$ Neglecting any multiple reflections/transmissions, determine the intensity of the light that is completely transmitted through the glass.

B. 3 points Unpolarized light is incident on a surface from above at $\theta = 45^o$. The index of refraction above the surface is $n_1 = 1.0$ and below it is $n_2 = 2.5$. What fraction of the reflected light intensity is polarized perpendicular to the plane of incidence.?

C. 3 points Compare transmitted intensities for one pane of glass of thickness $2.0cm$ and index $n = 2.0$ to a stack of two panes of the same glass of thickness $1.0\,cm$ each. Find the ratio of the intensity of light transmitted through the stack versus that transmitted through the single sheet of equal total thickness. Light is incident from air.