# Quantum and modern physics core examination

## 1

Consider Compton scattering of photons of energy $hf = 0.511\,MeV$. What will be the kinetic energy of the electron if the light is back-scattered (scattered through $\pi$ radians)?
**A.** $0.511 MeV$, **B.** $0\,MeV$, **C.** $0.170 MeV$, **D.** $340\,MeV$, **E.** $0.255\,MeV$.

## 2

Consider Compton scattering of photons of energy $hf = 0.511\,MeV$. What will be the kinetic energy of the electron if the light is scattered through $90^o$?
**A.** $0.511 MeV$, **B.** $0\,MeV$, **C.** $0.170 MeV$, **D.** $340\,MeV$, **E.** $0.255\,MeV$.

## 3

In a Hydrogen atomic transition what is the longest wavelength of light that can be absorbed by an electron in the ground state?
**A.** $97.2\,nm$, **B.** $102.5\,nm$, **C.** $121.5\,nm$, **D.** $213.7\,nm$, **E.** $375.89\,nm$

## 4

What is the group velocity of the wavepacket $\psi(x) = A\Big(\cos(\frac{8.0}{\mu m}x - \frac{4.0\times 10^2}{s}t) + \cos(\frac{7.6}{\mu m}x - \frac{3.98\times 10^2}{s}t)\Big)$?

## 5

What is the phase velocity of the wavepacket $\psi(x) = A\Big(\cos(\frac{8.0}{\mu m}x - \frac{4.0\times 10^2}{s}t) + \cos(\frac{7.6}{\mu m}x - \frac{3.98\times 10^2}{s}t)\Big)$?

## 6

Electrons of energy $E = 5.0\,eV$ have what de'Broglie wavelength?
**A.** $0.055\,nm$, **B.** $0.55\,nm$, **C.** $5.5\,nm$, **D.** $55.0\,nm$, **E.** $1.1\,nm$

## 7

A Helium atom has its electron in the ground state, with wavefunction $\Psi_0(r) = \frac{1}{\sqrt{\pi}}\sqrt{\frac{8}{a_0^3}}\,e^{-\frac{2r}{a_0}}$. The nucleus undergoes a sudden beta-decay ${}^4_2He \rightarrow {}^4_1 H + e^+ + \bar{\nu}_e$. What is the probability that the electron in the resulting heavy-Hydrogen will be in its ground state? The Hydrogenic ground state wavefunction is $\Psi_0(r) = \frac{1}{\sqrt{\pi}}\sqrt{\frac{1}{a_0^3}}\,e^{-\frac{r}{a_0}}$.

**8**

Consider electromagnetic waves trapped in a cubic box of side $a$. Approximately how many modes are trapped in the box with frequencies $\omega$ less than or equal to $10\,\frac{\pi c}{a}$?

**9**

For Black-Body radiation cavity radiation, if you triple the temperature, what happens to the wavelength of the light emitted from the cavity in greatest abundance?

**10**

A planar rotor, with Hamiltonian operator $\hat{H} = -\frac{\hbar^2}{2I}\frac{\partial^2}{\partial\phi^2}$ is prepared in a state with wavefunction $\Psi(\phi) = 0.5\frac{1}{\sqrt{2\pi}}e^{3i\phi} - \sqrt{\frac{3}{4}}\frac{1}{\sqrt{2\pi}}e^{-i\phi}$. What is the probability of an angular momentum measurement producing result $1\hbar$?
**A.** 0, **B.** $\frac{1}{4}$, **C.** $\frac{1}{2}$, **D.** $\sqrt{\frac{3}{4}}$, **E.** $\frac{3}{4}$.

**11**

What is the probability of an energy measurement producing result $\frac{\hbar^2}{2I}$?
**A.** 0, **B.** $\frac{1}{4}$, **C.** $\frac{1}{2}$, **D.** $\sqrt{\frac{3}{4}}$, **E.** $\frac{3}{4}$.

**12**

Which of the following operators are not Hermitean? Let $a$ be a real parameter.
**A.** $\hat{U} = e^{ia\frac{\partial}{\partial x}}$, **B.** $\hat{U} = ax$, **C.** $\hat{U} = ia\frac{\partial}{\partial x}$, **D.** $\hat{U} = -a^2\frac{\partial}{\partial x^2}$, **E.** $\hat{U} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

**13**

A planar rotor, with Hamiltonian operator $\hat{H} = -\frac{\hbar^2}{2I}\frac{\partial^2}{\partial\phi^2}$ is prepared in a state with wavefunction $\Psi(\phi) = \frac{1}{2}\frac{1}{\sqrt{2\pi}}e^{3i\phi} - \sqrt{\frac{3}{4}}\frac{1}{\sqrt{2\pi}}e^{-i\phi}$ at time $t = 0$. What is the wavefunction at a time $t$ later?
**A.** $\frac{1}{2}\frac{1}{\sqrt{2\pi}}e^{3i\phi} - \sqrt{\frac{3}{4}}\frac{1}{\sqrt{2\pi}}e^{-i\phi}$, **B.** $\frac{1}{2}\frac{1}{\sqrt{2\pi}}e^{3i\phi - i\frac{\hbar t}{2I}} - \sqrt{\frac{3}{4}}\frac{1}{\sqrt{2\pi}}e^{-i\phi - i\frac{\hbar t}{2I}}$, **C.** $\frac{1}{2}\frac{1}{\sqrt{2\pi}}e^{3i\phi - i\frac{3\hbar t}{2I}} - \sqrt{\frac{3}{4}}\frac{1}{\sqrt{2\pi}}e^{-i\phi - i\frac{\hbar t}{2I}}$, **C.** $\frac{1}{2}\frac{1}{\sqrt{2\pi}}e^{3i\phi - i\frac{9\hbar t}{2I}} - \sqrt{\frac{3}{4}}\frac{1}{\sqrt{2\pi}}e^{-i\phi - i\frac{\hbar t}{2I}}$, **E.** None of these.

**14**

An operator $\hat{U}$ is Hermitean if
**A.** $\langle \hat{U}\phi|\psi\rangle = \langle\phi|\hat{U}\psi\rangle$, **B.** $\langle(\hat{U}\phi)|(\hat{U}\phi\rangle = \langle\phi|\phi\rangle$, **C.** $\hat{U}^{-1} = \hat{U}^T$, **D.** $\hat{U}^{-1} = \hat{U}^\dagger$, **E.** None of these are the correct definition.

**15**

Which of the following wavefunctions is an eigenstate of the linear momentum operator?
**A.** $\psi_1 = A\sin kx$, **B.** $\psi_2 = Ae^{-ax^2}$, **C.** $\psi_3 = Ae^{ikx}$, **D.** $\psi_4 = A\sin^2 kx$, **E.** $\psi_5 = \lim_{\epsilon\to 0}\frac{1}{\pi}\frac{\epsilon}{x^2+\epsilon^2}$

**16**

Which of the following wavefunctions is an eigenstate of the one-dinensional position operator?
**A.** $\psi_1 = A\sin kx$, **B.** $\psi_2 = Ae^{-ax^2}$, **C.** $\psi_3 = Ae^{ikx}$, **D.** $\psi_4 = A\sin^2 kx$, **E.** $\psi_5 = \lim_{\epsilon\to 0}\frac{1}{\pi}\frac{\epsilon}{x^2+\epsilon^2}$

**17**

The probability distribution function for the outcome of position measurements is $\frac{d\wp(x)}{dx} = \frac{1}{\sqrt{\pi}\,\sigma}e^{-\frac{x^2}{\sigma^2}}$. For momentum measurements the distribution of outcomes is $\frac{d\wp'(p)}{dp} = \frac{1}{\sqrt{\pi}\,\sigma'}e^{-\frac{p^2}{(\sigma')^2}}$. In terms of $\sigma$, what is the minimal value of $\sigma'$?
**A.** $\sigma$, **B.** $\frac{\hbar}{\sigma}$, **C.** $\frac{\hbar}{2\sigma}$, **D.** $\frac{2\hbar}{\sigma}$, **E.** $\frac{\sigma}{\hbar}$

**18**

In Dirac bra-ket formalism, $|\phi\rangle$ is a ray, $\langle\phi|$ is its conjugate (dual) ray, and $\langle\phi|\psi\rangle$ is a complex number. What is $\sum_i a_i|\psi_i\rangle\langle\psi_i|$?
**A.** An operator, **B.** A ray, **C.** a dual ray, **D.** a complex number, **E.** a real number.

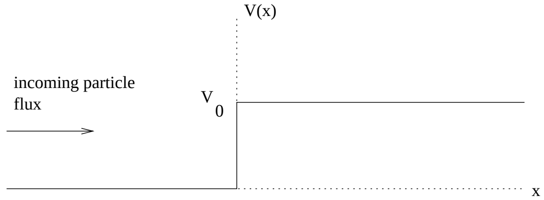
**19**

Consider a particle in a box $0 \le x \le L$, with states whose wavefunctions are $\psi_n(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$, with energies $E_n = \frac{\hbar^2 n^2\pi^2}{2mL^2}$. Suppose that a weak perturbation $V(x) = ax$ is present, by how much does it change the energies of these states?

**20**

The wavefunction of the Hydrogen ground state is $\psi(r) = \frac{1}{\sqrt{\pi a_0^2}}e^{-\frac{r}{a_0}}$. The potential it feels from the nucleus is $V(r) = -\frac{k}{r}$, $r \ge 0$. Suppose that the nucleus only exerted this potential for $r \ge a$, and for $r < a$ it exerted potential $\frac{-k}{a}$. By how much would the electronic energy change?

**21**




What is the correct form of the total wavefunction to the left of the origin for incident free particles of energy $E > V_0$? Let $p = \hbar k = \sqrt{2mE}$, $p' = \hbar k' = \sqrt{2m(E - V_0)}$
**A.** $e^{ikx}$, **B.** $Ae^{-|k'|x}$, **C.** $e^{-ik'x}$, **D.** $e^{ikx} + Ae^{-ikx}$, **E.** $Ae^{ik'x}$.

**22**

What is the correct form of the total wavefunction to the right of the origin for incident free particles of energy $E > V_0$? Let $p = \hbar k = \sqrt{2mE}$, $p' = \hbar k' = \sqrt{2m(E - V_0)}$
**A.** $e^{ikx}$, **B.** $Be^{-ikx}$, **C.** $e^{-ik'x}$, **D.** $e^{ikx} + Ae^{-ikx}$, **E.** $Ae^{ik'x}$.

**23**

What is the correct form of the total wavefunction to the right of the origin for incident free particles of energy $E < V_0$? Let $p = \hbar k = \sqrt{2mE}$, $p' = \hbar k' = \sqrt{2m(E - V_0)}$
**A.** $e^{ikx}$, **B.** $Be^{-ikx}$, **C.** $e^{-ik'x}$, **D.** $e^{ikx} + Ae^{-ikx}$, **E.** $Ae^{ik'x}$.

**24**

What is the correct form of the particle current density transmitted into the region to the right of the origin for $E > V_0$?
**A.** $\frac{\hbar k}{m}$, **B.** $\frac{\hbar k'}{m}$, **C.** $\frac{\hbar k}{m}|\frac{k-k'}{k+k'}|^2$, **D.** $\frac{\hbar k'}{m}|\frac{2k}{k+k'}|^2$, **E.** $\frac{\hbar k'}{m}|\frac{k-k'}{k+k'}|^2$

**25**

Which statement regarding the reflection coefficient in the case of $E < V_0$ is true?
**A.** Its amplitude is one, **B.** Its amplitude is less than one, but more than zero **C.** Its amplitude is more than one, **D.** It is zero, **E.** It is a real number.

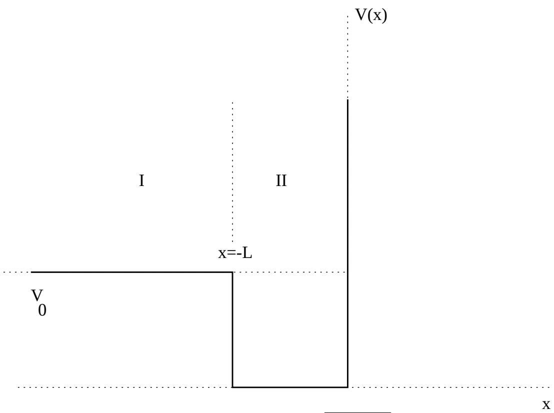
**26**

Consider a wavepacket of the form $\Psi(x,t) = \int \frac{1}{\sqrt{\pi}\sigma} e^{-\frac{(k_0-k')^2}{\sigma^2}} e^{ik'x - i\frac{\hbar(k')^2}{2m}t}\, dk'$. What is its group velocity?
**A.** $v_g = \frac{\hbar k_0}{m}$, **B.** $v_g = \frac{\hbar k'}{m}$, **C.** $v_g = \frac{\hbar k_0}{2m}$, **B.** $v_g = \frac{\hbar k'}{2m}$, **E.** $v_g = \frac{\hbar(k')^2}{2m}$

**27**

Consider a wavepacket of the form $\Psi(x,t) = \int \frac{1}{\sqrt{\pi}\sigma} e^{-\frac{(k_0-k')^2}{\sigma^2}} e^{ik'x - i\frac{\hbar(k')^2}{2m}t}\, dk'$. The packet collides with a complicated potential barrier, and the reflected wave packet looks like $\Psi_R(x,t) = \int \frac{1}{\sqrt{\pi}\sigma} e^{-\frac{(k_0-k')^2}{\sigma^2}} e^{ik'x - i\frac{\hbar(k')^2}{2m}t + 2i\phi(k')}\, dk'$. How long did the "particle" represented by the packet spend within the confines of the barrier before being expelled?
**A.** 0, **B.** $\frac{2}{v_g}\frac{d\phi(k_0)}{dk}$, **C.** $\frac{d\phi(k_0)}{dk}$, **D.** $t$, **E.** There is no way to know.

**28**




The figure shows a very simple potential often used to model nucleon-nuclear interparticle forces. In the two regions, for bound states $E < V_0$ we have

$$\psi_I = Be^{\mu x}, \quad \mu = \frac{\sqrt{2m(V_0 - E)}}{\hbar} = ik_I$$

and

$$\psi_{II} = A\sin(k_{II}x), \quad k_{II} = \frac{\sqrt{2mE}}{\hbar}$$

Which expression below determines the spectrum of bound state energies?

**A.** $\cot(\sqrt{2mE}\,\frac{L}{\hbar}) = -\sqrt{\frac{V_0-E}{E}}$, **B.** $\sin(\sqrt{2mE}\,\frac{L}{\hbar}) = -\sqrt{\frac{V_0-E}{E}}$, **C.** $\tan(\sqrt{2mE}\,\frac{L}{\hbar}) = \sqrt{\frac{V_0-E}{E}}$, **D.** $\sqrt{2mE}\,\frac{L}{\hbar}) = -\sqrt{\frac{V_0-E}{E}}$, **E.** $E$ could be anything between 0 and $V_0$.

**29**

Resonances for this potential are persistent standing waves created in region II for $E > V_0$. The amplitude of a resonance is

$$|A|^2 = \frac{4(\frac{E}{V_0} - 1)}{(\frac{E}{V_0} - 1) + \cos^2(\sqrt{\frac{2mEL^2}{\hbar^2}})}$$

Under what conditions placed on the $k = \frac{\sqrt{2mE}}{\hbar}$ wavenumber of the incoming wave will a resonance be created?
**A.** $kL = n\pi$, **B.** $kL = (n + \frac{1}{2})\pi$, **C.** $kL = 2n$, **D.** $kL = (2n + 1)$, **E.** $kL = 2n\pi$

**30**

What is the best description of the wavefunction in region II for a resonance?
**A.** Node at 0, node at $L$, **B.** Node at 0, antinode at $L$, **C.** Antinode at 0, node at $L$, **D.** Antinode at 0, antinode at $L$, **E.** None of these are accurate.

**31**

A resonance peak in the function $|A|^2$ given above has an approximate shape $|A|^2 \approx \frac{4}{(k-k_0)^2+\epsilon^2}$. What expression could determine the lifetime $\Delta t$ of the resonance described by this particular peak?
**A.** $2\epsilon\Delta t \geq \frac{\hbar}{2}$, **B.** $\frac{\hbar^2 k_0}{m}\Delta t \geq \frac{\hbar}{2}$, **C.** $2\epsilon\frac{\hbar^2 k_0}{m}\Delta t \geq \frac{\hbar}{2}$, **D.** $\epsilon^2\Delta t \geq \frac{\hbar}{2}$, **E.** $\frac{4}{(k-k_0)^2+\epsilon^2} \geq \frac{\hbar}{2}$

**32**

A resonance peak in the function $|A|^2$ given above has an approximate shape $|A|^2 \approx \frac{4}{(k-k_0)^2+\epsilon^2}$. Approximately what is the energy of the resonance described by this particular peak?
**A,** $\frac{\hbar^2 k_0^2}{2m}$, **B.** $V_0 + \frac{\hbar^2 k_0^2}{2m}$, **C.** $\frac{V_0}{2}$, **D.** $V_0 - \frac{\hbar^2 k_0^2}{2m}$, **E.** There is no way to compute this without the bound state spectrum.

**33**

Let the position operator be $\hat{q} = q$, and the momentum operator $\hat{p} = \frac{\hbar}{i}\frac{d}{dq}$. What is $[\hat{p}, \hat{q}^2] =$?
**A.** 0, **B.** $-i\hbar q$, **C.** $-2i\hbar q$, **D.** $-i\hbar\hat{p}$, **E.** $-2i\hbar\hat{p}$

**34**

The harmonic oscillator Schrodinger equation is $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi + \frac{1}{2}m\omega^2 x^2\psi = E\psi$, which can be rewritten using $\alpha = \frac{m\omega}{\hbar}$ and $q = \sqrt{\alpha}x$ and $\epsilon = \frac{2mE}{\hbar^2\alpha^2}$ as $(p^2 + q^2)\psi = \epsilon\psi$.. Define raising and lowering operators such that $a = \frac{1}{\sqrt{2}}(q + ip)$, $a^\dagger = \frac{1}{\sqrt{2}}(q - ip)$. What is $[a, a^\dagger] =$?
**A.** 0, **B.** 1, **C.** $-1$, **D.** $i$, **E.** $-i$

**35**

Which statement below is true of the ground state $\psi_0(q)$ of the harmonic oscillator?
**A.** $aa^\dagger\psi_0(q) = 0$, **B.** $(a^\dagger a + 1)\psi_0(q) = 0$, **C.** $a^\dagger\psi_0(q) = 0$, **D.** $a\psi_0(q) = 0$, **E.** $\psi_0(q) = 0$

**36**

Which statement below is the Schrodinger equation written in terms of the raising and lowering operators?
**A.** $aa^\dagger\psi(q) = \epsilon\psi(q)$, **B.** $aa^\dagger\psi(q) = (\epsilon - \frac{1}{2})\psi(q)$,**C.** $aa^\dagger\psi(q) = (\epsilon + \frac{1}{2})\psi(q)$, **D.** $aa^\dagger\psi(q) = \frac{1}{2}(\epsilon + 1)\psi(q)$, **E.** $aa^\dagger\psi(q) = \frac{1}{2}(\epsilon - 1)\psi(q)$

**37**

Which wavefunction below is the correct un-normalized ground state wavefunction?
**A.** $\psi_0(q) = Ne^{-q}$, **B.** $\psi_0(q) = Ne^{-q^2}$,**C.** $\psi_0(q) = Ne^{-\frac{q^2}{2}}$, **D.** $\psi_0(q) = Ne^{q^2}$,**E.** $\psi_0(q) = Ne^{\frac{q^2}{2}}$

**38**

What does $[\hat{p}_x, \hat{L}_x] =$

## 39

What does $[\hat{L}_x, \hat{L}_z] =$

## 40

What is the parity of the ground state harmonic oscillator wavefunction? Recall that $\mathcal{P}\psi(q) = 1 \cdot \psi(q)$ if $\psi(-q) = \psi(q)$ and $\mathcal{P}\psi(q) = -1 \cdot \psi(q)$ if $\psi(-q) = -\psi(q)$

## 41

Consider particles trappen in a box $-\frac{L}{2} \leq x \leq \frac{L}{2}$. Wave functions are $\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{2n\pi x}{L}$, and $\phi_n(x) = \sqrt{\frac{2}{L}} \cos \frac{(2n+1)\pi x}{L}$.
Consider a perturbation potential $V'(x) = ax$, $a > 0$. What is the first order shift in the energies the $\phi_2(x)$ state?

## 42

Consider a three state system $|a\rangle, |b\rangle, |c\rangle$ of respective energies $E_a < E_b < E_c$. Consider a perturbation of the system with potential $V'$, such that $\langle a|V'|a\rangle = 0$, $\langle b|V'|b\rangle = 0$, $\langle c|V'|c\rangle = \alpha$, $\langle a|V'|b\rangle = \beta$, $\langle a|V'|c\rangle = 0$, $\langle c|V'|b\rangle = 0$.
In first order non-degenerate perturbation theory, what will be the new energy spectrum?

## 43

Consider a three state system $|a\rangle, |b\rangle, |c\rangle$ of respective energies $E_a < E_b < E_c$. Consider a perturbation of the system with potential $V'$, such that $\langle a|V'|a\rangle = 0$, $\langle b|V'|b\rangle = 0$, $\langle c|V'|c\rangle = \alpha$, $\langle a|V'|b\rangle = \beta$, $\langle a|V'|c\rangle = 0$, $\langle c|V'|b\rangle = 0$.
In second order non-degenerate perturbation theory, what will be the new energy spectrum?

## 44

Consider a three state system $|a\rangle, |b\rangle, |c\rangle$ of respective energies $E_a = E_b = E_c = E_0$. Consider a perturbation of the system with potential $V'$, such that $\langle a|V'|a\rangle = 0$, $\langle b|V'|b\rangle = 0$, $\langle c|V'|c\rangle = \alpha$, $\langle a|V'|b\rangle = \beta$, $\langle a|V'|c\rangle = 0$, $\langle c|V'|b\rangle = 0$.
In degenerate-state perturbation theory, what will be the new energy spectrum?

## 45

Consider plane-waves $\psi_{in}(\mathbf{r}) = Ne^{i\mathbf{k}\cdot\mathbf{r}}$ incident on a Yukawa nuclear potential $V(r) = \frac{Ae^{-\sigma r}}{|\mathbf{r}|}$, in the Born approximation for elastic scattering;

$$\frac{d\sigma}{d\Omega} = \frac{m^2}{4\pi^2\hbar^2}\Big|\langle\psi_{in}|V(r)|\psi_{out}\rangle\Big|^2$$

what would the differential scattering cross-section be?

**46**

Consider plane-waves $\psi_{in}(\mathbf{r}) = Ne^{i\mathbf{k}\cdot\mathbf{r}}$ incident on a repulsive Coulomb potential $V(r) = \frac{A}{|\mathbf{r}|}$, in the Born approximation for elastic scattering;

$$\frac{d\sigma}{d\Omega} = \frac{m^2}{4\pi^2\hbar^2}\Big|\langle\psi_{in}|V(r)|\psi_{out}\rangle\Big|^2$$

what would the differential scattering cross-section be?

**47**

Fermi's Golden Rule for transition rates $|i\rangle \to |f\rangle$ caused by a perturbing potential $V$ is

$$R = \frac{2\pi}{\hbar}\Big|\langle f|V|i\rangle\Big|^2 \rho_f\, \delta(E_f - E_i)$$

Consider a simple quantum system with Schrodinger equation $\hat{H}_0\psi_n^0(x) = E_n^0\psi_n^0(x)$, state energies $E_n^0$. These stationary states evelve as $\psi_n^0(x,t) = \psi_n^0(x)\, e^{-i\frac{E_n^0}{\hbar}t}$. If we perturb the system $\hat{H}_0 \to \hat{H}_0 + \hat{V}(t)$, , the new wavefunctions are expandible in terms of these;

$$\psi(x,t) = \sum_n a_n(t)\psi_n^0(x)\, e^{-i\frac{E_n^0}{\hbar}t}$$

and insertion into the new Schrodinger equation results in

$$i\hbar\dot{a}_n = \sum_m \langle n|\hat{V}|m\rangle\, e^{-i\frac{E_m^0 - E_n^0}{\hbar}t}$$

If at $t = 0$ the system is in one of our unperturbed states, say the $k^{th}$, then $a_k(0) = 1$, $a_{n\neq k}(0) = 0$ and we find that

$$a_k(t) = 1 + \frac{1}{i\hbar}\int_0^t \langle k|\hat{V}(t')|k\rangle\, dt', \quad a_{j\neq k}(t) = \frac{1}{i\hbar}\int_0^t \langle j|\hat{V}(t')|k\rangle\, e^{-i\frac{E_k^0 - E_j^0}{\hbar}t}\, dt'$$

and we can find the transition probability $\wp = |a_j(t)|^2$ that the perturbation will make the system be in state $j$ in time $t$.
If the time dependence of the perturbation $V'(t0$ is simple that it is constant in time after $t = 0$, zero before $t = 0$, What is the probability $\wp_{i\to f}$ that if we began in state $i$ we end up in $f$ at time $t$?
**A.** $|\frac{2\langle f|H_I|i\rangle}{E_{fi}}|^2\sin^2(\frac{E_{fi}t}{2\hbar})$, **B.** $|\frac{2\langle f|H_I|i\rangle}{E_{fi}}|^2$, **C.** $|2\langle f|H_I|i\rangle|^2$, **D.** 0, **E.** 1.

**48**

If the density of states of energy $E$ is $\rho(E)$, the probability of eventually occupying a state with energy between $E_f$ and $E_f + dE_f$ is $\wp(t) = \wp_{i\to f}\,\rho(E_f)$, and the rate of this transition is $\frac{d\wp(t)}{dt} = R$. Show that the limit as $t \to \infty$ of this rate gives Fermi's Golden rule.

## 49

Consider a three state system $|a\rangle, |b\rangle, |c\rangle$ of respective energies $E_a < E_b < E_c$. Consider a perturbation of the system with potential $V'$, such that $\langle a|V'|a\rangle = 0$, $\langle b|V'|b\rangle = 0$, $\langle c|V'|c\rangle = \alpha$, $\langle a|V'|b\rangle = \beta$, $\langle a|V'|c\rangle = 0$, $\langle c|V'|b\rangle = 0$.
Suppose that the system begins in the state $|a\rangle$, and the potential $V'' = V' \sin\omega t$ is applied to it. What must $\omega$ be in order to stimulate transitions into state $c$?

## 50

Consider a three state system $|a\rangle, |b\rangle, |c\rangle$ of respective energies $E_a < E_b < E_c$. Consider a perturbation of the system with potential $V'$, such that $\langle a|V'|a\rangle = 0$, $\langle b|V'|b\rangle = 0$, $\langle c|V'|c\rangle = \alpha$, $\langle a|V'|b\rangle = \beta$, $\langle a|V'|c\rangle = 0$, $\langle c|V'|b\rangle = 0$.
Suppose that the system begins in the state $|a\rangle$, and the potential $V'' = V' \sin\omega t$ is applied to it. What must $\omega$ be in order to stimulate transitions into state $b$?

## 51

| Ket | N | l | $m_l$ | wavefunction |
|---|---|---|---|---|
| $\vert 100\rangle$ | 1 | 0 | 0 | $\frac{1}{\sqrt{\pi}}(\frac{Z}{a_0})^{\frac{3}{2}}e^{-\frac{Zr}{a_0}}$ |
| $\vert 200\rangle$ | 2 | 0 | 0 | $\frac{1}{4\sqrt{2\pi}}(\frac{Z}{a_0})^{\frac{3}{2}}(2-\frac{Zr}{a_0})e^{-\frac{Zr}{2a_0}}$ |
| $\vert 210\rangle$ | 2 | 1 | 0 | $\frac{1}{4\sqrt{2\pi}}(\frac{Z}{a_0})^{\frac{3}{2}}\frac{Zr}{a_0}\cos\theta e^{-\frac{Zr}{2a_0}}$ |
| $\vert 21\pm1\rangle$ | 2 | 1 | $\pm1$ | $\frac{1}{8\sqrt{\pi}}(\frac{Z}{a_0})^{\frac{3}{2}}\frac{Zr}{a_0}\sin\theta e^{-\frac{Zr}{2a_0}}e^{\pm i\phi}$ |

In the Zeeman effect, the energy levels of an electron in an atom are shifted by a magnetic field, $\hat{V}' = -\boldsymbol{\mu}\cdot\mathbf{B}$. Suppose that $\mathbf{B} = B_0\mathbf{k}$. If the electronic gyromagnetic ratio is $g$, find the new energies of the states $|200\rangle, |210\rangle, |211\rangle, |21-1\rangle$ in first order perturbation theory.

## 52

In the Stark effect, the current of the electron interacts with an external electric field. Electrical current of an electron is $\mathbf{J} = -\frac{e\hbar}{m} Im\left(\psi\boldsymbol{\nabla}\psi^*\right)$. Which states should experience no first order perturbation (which states have no electrical current)?

## 53

In the Stark effect, the perturbing potential is $\hat{V} = eE_0 r\cos\theta$. In degenerate perturbation theory, which of the states $|200\rangle, |210\rangle, |211\rangle, |21-1\rangle$ will be "mixed" by the perturbation to form the new perturbed states? In other words find those pairs of states $\Psi_i\rangle$, $\Psi_j\rangle$ such that $\langle\Psi_i|\hat{V}|\Psi_j\rangle \neq 0$.

## 54

10 electrons are placed in a box of dimensions $L \times L \times L$. What is the Fermi energy (energy of the uppermost occupied level) at $T = 0$ (no thermal excitation)?

## 55

Apply the Aufbau process and determine the electron structure symbol for sodium (Example; He has symbol $1s^2$).

## 56

An electron "orbiting" a nucleus creates an electrical current that in turn creates a magnetic field that its spin angular momentum interacts with. The interaction is $\hat{V} = \mu_B \mu_s \mathbf{L} \cdot \mathbf{s}$. Which of the following operators commutes with this addition to the Hamiltonian?
**A.** $\mathbf{L}$, **B.** $\mathbf{S}$, **C.** $\mathbf{J}^2$, **D.** $\mathbf{L}^2$, **E.** $\mathbf{S}^2$

## 57

An electron "orbiting" a nucleus creates an electrical current that in turn creates a magnetic field that its spin angular momentum interacts with. The interaction is $\hat{V} = \mu_B \mu_s \mathbf{L} \cdot \mathbf{s}$. Find the energy shift in the states $2S^2_{\frac{1}{2}}, 2S^2_{\frac{3}{2}}, 2P^2_{\frac{1}{2}}, 2P^2_{\frac{3}{2}}$ in first order non-degenerate perturbation theory.

## 58

Denote by $\alpha$ the spin state $|\frac{1}{2}, \frac{1}{2}\rangle = |s, m_s\rangle$, and $\beta = |\frac{1}{2}, -\frac{1}{2}\rangle$. From these find the wavefunction of a state $s = 1$, $m_s = 0$.
**A.** $\alpha \otimes \beta$, **B.** $\alpha \otimes \alpha$,**C.** $\frac{1}{\sqrt{2}}(\alpha \otimes \beta + \beta \otimes \alpha)$, **D.** $\frac{1}{\sqrt{2}}(\alpha \otimes \beta - \beta \otimes \alpha)$, **E.** $\beta \otimes \beta$.

## 59

Denote by $\psi_{i,\uparrow}(k)$, $\psi_{i,\downarrow}(k)$, spin up/down electron number $k$ in energy level $E_i$. Which wavefunction is correct for a state of energy $2E_1$?
**A.** $\psi_{1,\uparrow}(1) \otimes \psi_{1,\uparrow}(2)$, **B.** $\psi_{1,\uparrow}(1) \otimes \psi_{1,\downarrow}(2)$, **C.** $\frac{1}{\sqrt{2}}\Big(\psi_{1,\uparrow}(1) \otimes \psi_{1,\uparrow}(2) + \psi_{1,\uparrow}(2) \otimes \psi_{1,\uparrow}(1)\Big)$, **D.** $\frac{1}{\sqrt{2}}\Big(\psi_{1,\uparrow}(1) \otimes \psi_{1,\uparrow}(2) - \psi_{1,\uparrow}(2) \otimes \psi_{1,\uparrow}(1)\Big)$, **E.** $\frac{1}{\sqrt{2}}\Big(\psi_{2,\uparrow}(1) \otimes \psi_{1,\uparrow}(2) - \psi_{1,\uparrow}(2) \otimes \psi_{2,\uparrow}(2)\Big)$

## 60

Two spin-$\frac{1}{2}$ particles interact via Hamiltonian $\hat{H} = A\mathbf{s}_1 \cdot \mathbf{s}_2$, what are the energy levels of stationary states?

## 61

A spin-$\frac{1}{2}$ particle interacts with a magnetic field $\mathbf{B} = B_0\hat{k}$ via $\hat{H} = -\mu\mathbf{B}\cdot\mathbf{s}$. If the state of the system is $|\frac{1}{2},\frac{1}{2}\rangle$ at $t = 0$, what is the probability that it will be $|\frac{1}{2},-\frac{1}{2}\rangle$ at $t$?

## 62

A particle in a box $0 \leq x \leq L$ is in its ground state. The wall is suddenly moved to $2L$. What is the probability that the system will remain in the ground state?

## 63

Consider massless, spin one-half particles that obey the Weyl equation

$$H\psi = -i\hbar c\boldsymbol{\nabla}\cdot\boldsymbol{\sigma}\psi = i\hbar\frac{\partial}{\partial t}\psi, \quad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

in which $\sigma_i$ are the Pauli spin matrices. Determine

$$[H, \mathbf{p}] = [H, -i\hbar\boldsymbol{\nabla}], \quad [H, \mathbf{L}]$$

in which $\mathbf{L} = \mathbf{r}\times\mathbf{p}$ is the orbital angular momentum vector.